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Mapping the Complexity of Democracy: Entropy-Driven Regime Discovery with Machine Learning and Explainable AI

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Abstract

Democracy is a multifaceted institutional phenomenon that cannot be comprehensively captured by a single index. This paper defines unique institutional configurations in various countries according to six V-Dem democracy dimensions, including the Liberal Democracy Index, Electoral Democracy Index, Liberal Component Index, Egalitarian Component Index, Participatory Component Index, and Deliberative Component Index. It uses data on 179 countries and conducts a primary K-means clustering method, supplemented with Hopkins statistics and gap statistics, bootstrap stability tests, and Gaussian Mixture Model (GMM) robustness tests. The paper also uses Shannon entropy with Miller–Madow correction to measure a regime's institutional heterogeneity, and the Rule of Law Index of the World Justice Project for external validation. As a result, the paper uncovers seven empirically identifiable democracy regimes featuring different institutional configurations. Different regimes have varying levels of internal institutional heterogeneity, being higher in hybrid regimes, lower in consolidated regimes. WJP analysis reveals additional systematic differences in rule of law outcomes in different regimes. Overall, the results are in line with the multidimensional approach to democracy and show how unsupervised learning and information-theoretic analysis may complement democracy indices.

Keywords: democracy regimes; complex adaptive systems; shannon entropy; machine learning; explainable artificial intelligence

1. Introduction

The level of democracy has been traditionally evaluated using aggregated measures that place nations on a democratic to authoritarian spectrum. Nevertheless, modern democracies vary greatly in terms of institutional arrangements. The development of electoral competition, liberalism, participation, equality, and deliberation can be unbalanced, leading to various types of democracy that cannot be comprehensively described using a single measure.

Previous studies have considered the multilayered nature of democracy. Dahl's (2008) [1] approach to the study of polyarchy emphasized the importance of the involvement of citizens and pluralism, and later research gave more emphasis to institutions, freedoms, and accountability [2]. Additionally, it was revealed by studying hybrid regimes and competitive authoritarianism that many political systems combine elements of democracy and authoritarianism [3,4].

The V-Dem project offers an extensive conceptualization for the investigation of democracy through several different dimensions: electoral, liberal, participatory, deliberative, and

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egalitarian [5–7]. Nevertheless, a very important question arises: do countries constitute specific regimes of democracy depending on multidimensional features or, instead, do they represent merely various points on a continuous line?

With the help of modern machine learning technologies, there is an opportunity to uncover latent institutional structures. With the use of unsupervised learning approaches, it is possible to reveal hidden patterns in the datasets without applying any prior classifications. At the same time, the results of the clustering need to be validated, since the statistical groups do not necessarily reflect political regimes.

In addition to recognizing democratic regimes, this study employs an information-theory approach to analyze institutional complexity. Conventional democracy indicators concentrate mainly on the average level of performance, while entropy-based analysis focuses on institutional heterogeneity. Entropy analysis allows one to measure diversity and uncertainty in complex systems [8].

Therefore, this study is aimed at answering the following research questions:

(1) Can multidimensional democracy indicators identify institutional regimes?
(2) Are the identified regimes stable against different clustering algorithms and validation techniques?
(3) Is there any difference between democratic regimes in terms of institutional complexity?

This study advances the existing research agenda by means of building a data-driven democracy typology based on V-Dem indicators, employing rigorous unsupervised clustering validation and entropy-based analysis.

2. Literature Review

The idea of measuring democracy has traditionally been viewed as a continuum of institutions on which states would be rated according to their democratic performance. The classical measures of democracy as presented in the works of Robert A. Dahl, Arend Lijphart, and Freedom House have contributed much to the science of comparative politics because they provide an objective measurement of the level of democracy. However, recent scholarship has shown that democracy cannot be measured as a continuum, since states have both democratic and authoritarian institutions and have different mixes of these institutions rather than identical levels of democratic development [2,3].

The rise of regimes such as hybrids, competitive authoritarians, and electoral authoritarians poses additional challenges to traditional democracy categorizations. For example, Larry Diamond came up with the idea of hybrid regimes, where the regime is characterized by democratic processes combined with authoritarian features [2]. Later, Levitsky and Lucan A. Way proved that competitive authoritarian regimes have an institutional structure that cannot be captured by binary democratic–authoritarian classification [3]. Most recently, the Regimes of the World classification developed in the Varieties of Democracy Institute provides a multidimensional classification of four regime types—liberal democracies, electoral democracies, electoral autocracies, and closed autocracies—through several institutional dimensions and not democracy scores [9].

Unlike traditional measures of democracy, the V-Dem index defines democracy as a multidimensional institutional construct encompassing five dimensions, namely electoral, liberal, egalitarian, participatory, and deliberative. This definition reflects the fact that democratization is an unequal process between nations, and different institutional components can develop at different levels [5,6].

Recent developments in machine learning techniques provide an option for discovering hidden institutional structures without defining any predetermined boundaries of regimes in advance. Unsupervised learning techniques, especially K-means and Gaussian Mixture Models, have been increasingly used in social sciences to discover patterns and

classify observations according to multidimensional similarity rather than to theory [10,11]. However, methodological literature stresses the importance of clustering analysis being validated in accordance with criteria such as cluster tendency analysis, bootstrap validation, consensus clustering, and different clustering algorithm comparisons [12–14].

A further problem with existing classifications of democratic systems is that they emphasize the average performance of institutions at the cost of neglecting the diversity of institutions within each regime category. An alternate view from information theory can be used to capture structural diversity using entropy indices. Shannon entropy represents the diversity of a probability distribution [8], whereas Miller–Madow correction accounts for sample size bias in entropy estimation [15]. Recent advances in statistics have emphasized the importance of entropy estimators with bias corrections for small samples [16]. Relative to variance, which is a traditional measure of diversity, entropy considers the multidimensional nature of institutions.

However, while these advances have been made, only a few papers consider the use of multidimensional democracy measurement, unsupervised learning techniques, cluster validation, and measures of information-theoretic complexity in an integrated manner. The current paper seeks to fill this void in the literature using techniques such as clusterability measures, multiple clustering algorithms, bootstrap-based robustness, external validation using institutions, and entropy-based complexity analysis.

3. Materials and Methods

The current paper establishes an analytic framework for exploring the extent to which global democracies can be categorized as institutional regimes with respect to their democratic institutions. Instead of considering democracy one-dimensional, the proposed framework is able to uncover underlying configurations of institutions based on multidimensionality through unsupervised learning, along with a statistical, information theory, and external approach to validate such regimes. The general research design and analytic framework are shown in Figure 1 below.

The process of analyzing is comprised of five interrelated steps. As seen in Figure 1, the first step is getting multidimensional democracy indicators from the Varieties of Democracy (V-Dem) data set and translating them into an analyzable matrix. The second step includes a clustering analysis to find latent democracy regimes by means of unsupervised clustering algorithms such as K-means, where the number of clusters is determined via multiple criteria (elbow analysis, silhouette coefficient, gap statistic). The third step is assessing the stability of the discovered regimes based on independent validation techniques, including the Hopkins cluster tendency test, a bootstrapping approach with Jaccard similarity, consensus clustering, and GMM comparison. The fourth step is measuring the internal heterogeneity and complexity of regimes by means of Shannon entropy with Miller–Madow correction. The fifth step is external validation with the help of the World Justice Project Rule of Law Index and explainable machine learning techniques for estimating the contribution of different dimensions of democracy to the Liberal Democracy Index prediction.

The fifth step includes external validation based on the World Justice Project Rule of Law Index along with explainable machine learning predictions of the Liberal Democracy Index. Given that the LDI is calculated based on two components, the Electoral Democracy Index and the Liberal Component Index, via a specific procedure of index aggregation, predicting the LDI based on those components would involve a mathematically defined dependence between them. Hence, the good predictive ability and the importance of those two indexes in the analyses of feature importance are at least partially expected and should not be interpreted as an additional discovery of institutional dependencies.

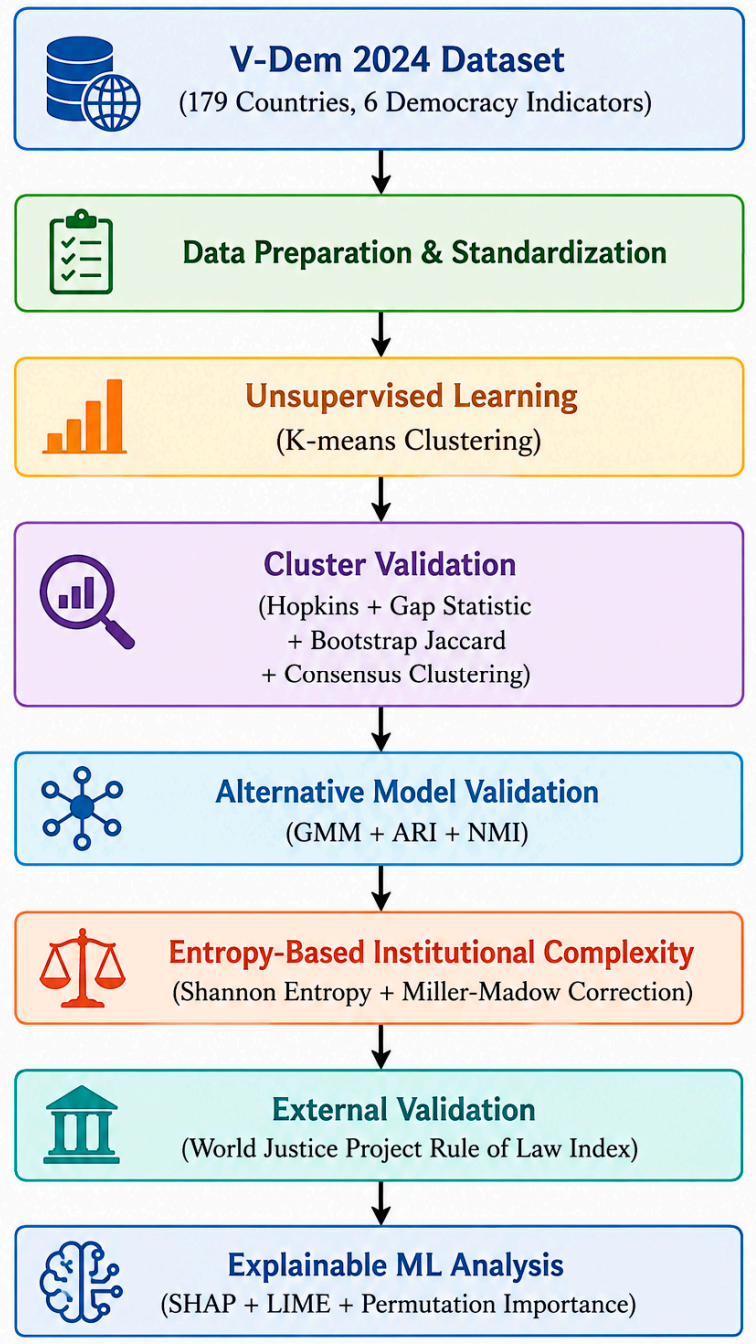

Figure 1. Analytical Workflow of the Machine Learning Pipeline for Democracy Classification.

3.1. Dataset and Democracy Indicators

The paper presents an analysis of multidimensionality of global democracy based on information collected within the frame of the Varieties of Democracy (V-Dem) Project, namely, the Democracy Report 2025 and V-Dem Dataset Version 2024. The database provides yearly measures of democracy for 179 countries for the year 2024 and uses a comprehensive and large-scale expert-coded framework of measurement. Unlike traditional measures of democracy that emphasize only electoral competition, the V-Dem framework views democracy as a multidimensional institutional system composed of electoral, liberal, participatory, egalitarian, and deliberative elements [5].

In empirical research, six fundamental democracy indicators are used to assess the multidimensionality of democratic institutions: the Liberal Democracy Index (LDI), Electoral Democracy Index (EDI), Liberal Component Index (LCI), Egalitarian Component Index

(ECI), Participatory Component Index (PCI), and Deliberative Component Index (DCI). These indicators together reflect the major institutional mechanisms of democratic governance, which include electoral competition, constraints of institutions, political inclusion, equality, and deliberative capacity. All the indicators are normalized by V-Dem and have a score ranging from 0 to 1, where a larger score implies better performance of democracy.

The database can be described by means of country by indicator matrix:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{16} \\ x_{21} & x_{22} & \cdots & x_{26} \\ \vdots & \vdots & \ddots & \vdots \\ x_{179,1} & x_{179,2} & \cdots & x_{179,6} \end{bmatrix} \quad (1)$$

In Equation (1), each row i represents a country ($i = 1 \ldots 179$), and each column j represents one of the six democracy dimensions ($j = 1 \ldots 6$). Therefore, each country is characterized by a six-dimensional institutional profile:

$$X_i = (LDI_i, EDI_i, LCI_i, ECI_i, PCI_i, DCI_i) \quad (2)$$

Prior to performing statistical analysis, several data pre-processing techniques were used. These include data cleaning, variable verification, checking missing values, and classification of countries based on regions using Python 3.13. As distance-based clustering and machine learning models are scale sensitive, all democracy variables were standardized using Z score normalization using Formula (3) [17]:

$$z_{ij} = \frac{x_{ij} - \overline{x}_j}{s_j} \quad (3)$$

In Equation (3), x_{ij} represents the value of democracy indicator j for country i, $\overline{x}_j$ denotes the mean value of indicator j, and s_j represents the corresponding standard deviation.

Consequently, the standardized matrix is as in Equation (4).

$$\boldsymbol{Z} = \left[z_{ij}\right]_{179\times 6} \quad (4)$$

This generated data was used in turn as the input for all the empirical analyses, including unsupervised clustering, GMM estimation, cluster robustness analysis, entropy institutional complexity analysis, external validation, and XML modeling.

Standardization means that all dimensions of democracy carry the same weight in the analytical approach and that measures that have greater numerical variance do not dominate the process of calculating distances or building predictive models. Thus, the framework assesses democracy as an institutionally multidimensional construct, rather than a single ranking number.

3.2. K-Means Clustering

The determination of democracy regimes was first carried out through the K-means clustering algorithm. This unsupervised learning technique involves partitioning the countries into K clusters through the minimization of variance and maximization of similarity among the countries with similar institutions [10,18–22].

The K-means objective function can be expressed mathematically as:

$$\min_{C_1,\ldots,C_k} W_k = \sum_{k=1}^{K} \sum_{x_j \in C_k} \|\boldsymbol{x}_i - \boldsymbol{\mu}_k\|^2 \quad (5)$$

where C_k is the cluster k, $\boldsymbol{x}_i$ is the democracy indicator vector of country i, and $\boldsymbol{\mu}_k$ is the corresponding centroid of the cluster.

The optimal number of clusters was identified based on several methods, including among others, the elbow criterion, silhouette score, and gap statistic. The silhouette score assesses intra-cluster cohesion and inter-cluster separation:

$$S(i) = \frac{b(i) - a(i)}{\max(a|(i)b(i))} \tag{6}$$

$a(i)$ is the average distance within the assigned cluster, while $b(i)$ is the lowest average distance to other clusters in Equation (6). The higher the value, the better defined the cluster [23,24].

Furthermore, the gap statistic measures the within-cluster dispersion against a null reference distribution:

$$Gap(K) = E^*[\log(W_K)] - \log(W_K) \tag{7}$$

In Equation (7) W_K represents the within cluster dispersion while E^* presents the expected value under the reference distribution [25]. The cluster solution was ultimately determined through a combination of statistical and stability criteria as well as interpretability.

3.3. Gaussian Mixture Model and Alternative Latent Structures

The robustness of the democracy regimes mentioned above is measured using the Gaussian Mixture Model (GMM) as a probabilistic clustering method. In contrast to K-means, the GMM model measures the likelihood of observations falling under different regimes and has no assumptions regarding equal covariance structure across clusters [26].

Gaussian mixture models refer to a weighted combination of the M components of Gaussian densities expressed by the following Equation (8).

$$\mathcal{P}(x \setminus \boldsymbol{\lambda}) = \sum_{i=1}^{M} \omega_i \, g(\boldsymbol{x} \mid \mu_i, \, \boldsymbol{\Sigma}_i) \tag{8}$$

Here, $\boldsymbol{x}$ is a D-dimensional vector of continuous-values, ω_i, $i = 1, \ldots, M$ are the mixture weights, and $g(\boldsymbol{x}|\mu_i, \Sigma_i)$, $i = 1, \ldots, M$ are the Gaussian densities [26].

Optimal GMM model specification was determined based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC):

$$AIC = -2\log(L) + 2p \tag{9}$$

$$BIC = -2\log(L) + p\log(n) \tag{10}$$

where L is the model likelihood, p is the number of estimated parameters, and n is the sample size. A lower AIC and BIC score implies a better-fitting model after accounting for model complexity [27,28].

The level of correlation in the clustering outputs from K-means and GMM was quantified using the Adjusted Rand Index (ARI) and the Normalized Mutual Information (NMI). The ARI stands for the chance-adjusted version of the Rand Index and is given by the following expression [29,30]:

$$ARI = \frac{RI - E(RI)}{1 - E(RI)} \tag{11}$$

In Equation (11), RI stands for Rand index and E(RI) is the expected value of this index under the assumption of the chance model given the observed size distribution of the clusters. ARI can take any value up to 1, and the higher it is, the better the agreement between the two partitions is; values close to zero mean an agreement which could result from chance [31].

3.4. Cluster Validation and Stability Analysis

For the testing of the stability of the obtained democracy regimes, different approaches to the cluster validation were employed. The Hopkins statistics were applied to check whether the data show any sign of spatial regularity or clustering, or are completely spatially random. The statistics are given by [32,33]:

$$H = \frac{\sum_{i=1}^{m} u_i^d}{\sum_{i=1}^{m} u_i^d + \sum_{i=1}^{m} w_i^d} \tag{12}$$

In Equation (12), u_i is the distance between a point and its nearest observation, w_i is the nearest distance among observed points, and d is the dimension of the dataset.

Furthermore, cluster stability was assessed using bootstrap stability analysis, which uses 1000 replications to check whether the clustering is consistent on resampling. The Jaccard similarity coefficient was used to measure cluster stability [34].

$$J(A|B) = \frac{|A \cap B|}{|A \cup B|} \tag{13}$$

In Equation (13), A and B are membership vectors corresponding to clusters formed on the original and bootstrap datasets, respectively [12,13]. Finally, consensus clustering was carried out to investigate the stability of the country-wise classifications in the bootstrap replicates.

Thus, the consensus matrix M can be defined as a well-normalized sum of the connectivity matrixes of all the perturbed datasets $\{D(h)$: $h = 1, 2,\ldots, H\}$:

$$M(i,j) = \frac{\sum_h M^{(h)}(i,j)}{\sum_h I^{(h)}(i,j)} \tag{14}$$

This implies that the element (i, j) of the consensus matrix will store the frequency of the co-clustering of items i and j divided by the overall frequency of the choice of both items. The symmetry of the consensus matrix should be obvious, since $M(i,j) = M(j,i)$ for all i and j. The element (i, j) of the consensus matrix will be called the consensus index for the respective pair of items [35].

3.5. Entropy-Based Institutional Complexity Analysis

Alongside the primary regime classification, Shannon entropy is applied to assess the institutional diversity of the six democracy dimensions of V-Dem (LDI, EDI, LCI, ECI, PCI, and DCI) in each democracy regime previously found. As the indicators were scaled between [0, 1], three institutional regimes (low, medium, and high) are defined with the help of thresholds 1/3 and 2/3.

For regime r, dimension j, and institution c, the probability of each state is determined as follows:

$$p_{rjc} = \frac{n_{rjc}}{n_r},\ c = 1, \ldots, K \tag{15}$$

In Equation (15), n_{rjc} is the number of countries in regime c for dimension j in regime r, and n_r is the total number of countries in regime r. Here, $K = 3$.

The Shannon entropy for each dimension of democracy can be calculated using the following formula:

$$H_{rj} = -\sum_{c=1}^{K} p_{rjc} \ln\left(p_{rjc}\right) \tag{16}$$

where p_{rjc} denotes the share of countries belonging to institutional state c. The total entropy for the regime r is calculated by averaging the entropy for all six dimensions of democracy:

$$H_r = \frac{1}{J}\sum_{j=1}^{J} H_{rj},\ J = 6 \tag{17}$$

In order to account for finite sample bias, the Miller–Madow correction is implemented [15,16]:

$$H_r^{MM} = H_r + \frac{K-1}{2n_r} \tag{18}$$

After that, the corrected entropy is normalized by the maximal possible entropy, ln(K), to calculate the Complexity Index:

$$CI_r = \frac{H_r^{MM}}{max_s \in R(H_s^{MM})} \tag{19}$$

In Equation (19), R is the collection of seven democracy regimes. A regime having the largest corrected average entropy will be allocated a Complexity Index value of 1.0000, whereas other regimes will get proportionately lower values. It is noteworthy that the above normalization is based on the largest observed entropy rather than the theoretical entropy maximum of $ln(K)$, which is 1.653597. Hence, the Complexity Index is a relative measure of institutional complexity, as it is relative to the observed rather than the theoretical maximum entropy.

CI is larger in cases when there is more diversity in the regime r in institutional structure, which leads to heterogeneity in the allocation of countries among the three institutional states.

The Complexity Index is the arithmetic mean of the entropy of the predefined institutional states for all democracy indicators in each cluster. The entropy of each indicator is derived based on the distribution of states across the countries for each indicator, which are then averaged to arrive at a measure of complexity for the cluster. Larger values imply a higher degree of heterogeneity in the distribution of institutional states among the countries in the cluster, while smaller values imply a higher degree of homogeneity in terms of the number of states. This index is hence indicative of a certain kind of institutional heterogeneity among the countries in a particular cluster. It does not represent any balance in the institutional characteristics of the countries, nor does it reflect any dependence/correlation between the democracy indicators.

It is important to notice that entropy is not used as another method of clustering. Seven democracy regimes were determined beforehand by means of K-means clustering and consensus clustering validation and further entropy analysis was conducted within these regimes. Thus, entropy is not a criterion of regime determination or reclassification. The values of the corrected average entropy and Complexity Index for seven democracy regimes are presented in Table 3.

The assignment of the institutional indicators to three categories (low, medium, and high) relies upon cutoffs at the 1/3 and 2/3 points. While these thresholds allow a clear and unambiguous discretization of the [0, 1] range, they should not be taken as universal thresholds distinguishing different types of institutions. In consequence, the entropy values incorporate both the distribution of the countries between different institutional states and the specific discretization procedure employed. Hence, the Complexity Index reflects institutional-state diversity among the democracy regimes discovered earlier, not a universal measure of institutional complexity. The validity of the main conclusions should thus be evaluated through investigating the sensitivity of the relative orderings and interpretations of the seven democracy regimes to the other thresholds.

3.6. External Validation of Democracy Regimes

The identified democracies were validated against the World Justice Project Rule of Law Index to see whether there are any differences between them beyond the pattern of clustering. Differences between the identified regimes were tested using one-way ANOVA [36]:

$$F = \frac{MS_{between}}{MS_{within}} \tag{20}$$

In Equation (20), $MS_{between}$ and MS_{within} represent the between-groups and within-group variance components. The degree of variation among the regimes was also measured by eta squared [37–39]:

$$\eta^2 = \frac{SS_{factor}}{SS_{total}} \tag{21}$$

In Equation (21), SSB is the sum of squares between groups and SST is the total sum of squares.

This measure allows us to estimate the proportion of variance of the external variable associated with differences between the regimes. The reporting of effect size for ANOVA is encouraged as a complement to hypothesis testing [38,40].

Most importantly, the rule of law index has not been used as one of the variables used in the clustering process. The WJP rule of law index can therefore serve as an independent criterion for checking whether there are differences in the institutional outcome.

3.7. Explainable Machine Learning Framework

The explainable machine learning approach was employed to explore the structure of predictions about liberal democracy. The purpose of the exercise was not the determination of causal factors but rather the contribution of various dimensions of democracy to the prediction of LDI.

The problem of prediction was formulated as:

$$LDI = f(EDI, LCI, ECI, PCI, DCI) + \epsilon \tag{22}$$

Regression algorithms were tested for a complete set of regression algorithms, which included linear algorithms (Linear Regression, Ridge, Lasso, Elastic Net, Bayesian Ridge, Huber Regressor, Passive Aggressive Regressor, Stochastic Gradient Descent Regressor, and RANSAC), distance and kernel-based techniques (K-Nearest Neighbors, Support Vector Regression, and NuSVR), tree-based and ensemble learning algorithms (Decision Tree, Random Forest, Extra Trees, AdaBoost, Gradient Boosting, and Bagging), more advanced boosting techniques (XGBoost, LightGBM, and CatBoost), and one type of Artificial Neural Networks (Multi-Layer Perceptron, MLP). The models were analyzed based on R^2, RMSE, and MAE measures.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \overline{y})^2} \tag{23}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{24}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \tag{25}$$

The interpretation of the model used the SHAP values approach, where each prediction is explained by decomposing it into contributions from individual features [41,42]:

$$f(x) = \phi_0 + \sum_{i=1}^{M}\phi_i \tag{26}$$

Here, ϕ_i refers to the contribution of feature i to the prediction. Also, other robustness checks through LIME and permutation approaches were used [17,41–44].

4. Empirical Findings and Model Performance Analysis

The current section outlines empirical results regarding the multidimensional nature of democracy around the globe using six V-Dem (2024) democracy indices. Instead of conceptualizing democracy as one-dimensional construct which varies from authoritarianism to democracy, democracy is seen as a complex institutional system made up of many dimensions such as electoral competitiveness, liberal constraints, participation, equality, and deliberation. Empirical analysis involves four main steps. First, the global variation in democracy indices is analyzed. Second, latent democratic regimes are identified using unsupervised machine learning methods. Third, the clusters' robustness and validity are analyzed using several diagnostic tools and different clustering algorithms. Fourth, the institutional attributes of democratic regimes are explored.

4.1. Global Democracy Patterns and Multidimensional Analysis of Democracy Indicators

Differences in levels of democratic development between countries were investigated through an examination of how the LDI was distributed in each country included in the V-Dem dataset. Figure 2 shows the twenty countries with the highest and lowest LDI.

This classification shows significant differences in the performance of global democracy. Those nations that are situated higher in terms of the LDI scale tend to have higher institutional constraints, competitive elections, and more individual rights protection. In contrast, those countries that have low LDI ratings show limited electoral competitiveness, lower institutional responsibility, and limited civil liberties. There is a significant difference between the top- and bottom-ranked countries, which shows that democracy is quite diverse.

However, democracy cannot be properly described by one index, because some of the institutional dimensions can develop at different rates. That is why the spatial distribution of the six V-Dem measures—LDI, EDI, LCI, ECI, PCI, and DCI—were considered. Global spatial patterns of the above-mentioned indicators are shown in Figure 3.

The global maps reveal significant differences about democratization along various dimensions. While some countries have uniformly high scores on many measures, other countries reveal mixed institutional performance. There are some countries that may have relatively competitive electoral processes but also low liberal or participatory measures. Such findings clearly indicate that democracy is an institutional system made up of different dimensions.

Descriptive statistics of democracy indicators also illustrate the discrepancies mentioned. The mean value of LDI is quite restricted (0.375), which implies that liberal democracies are still prevalent only in a rather narrow set of countries. Meanwhile, EDI illustrates an average value that is larger (0.486), implying that the prevalence of electoral institutions is higher compared to the overall prevalence of liberal democracies. In addition, the Liberal Component Index is characterized by a relatively high mean value (LCI = 0.589), illustrating that some degree of institutional constraints exists even in the countries where a greater democratic quality cannot be observed. Finally, the Deliberative Component Index reveals a relatively high mean value (DCI = 0.583), whereas the Participatory Component Index reveals a moderate world value (PCI = 0.470).

This shows that democratic institutions develop differently in different nations. Nations can be successful in some democratic dimensions like electoral process but still lag behind in other dimensions such as the liberal, participation, or deliberative dimensions. Thus, analyzing democracy as multiple dimensions will give us a better understanding of institutional variations.

Geographical variations were analyzed using the calculation of the mean score of democracy in each continent. Geographical variations in six dimensions of democracy are given in Figure 4.

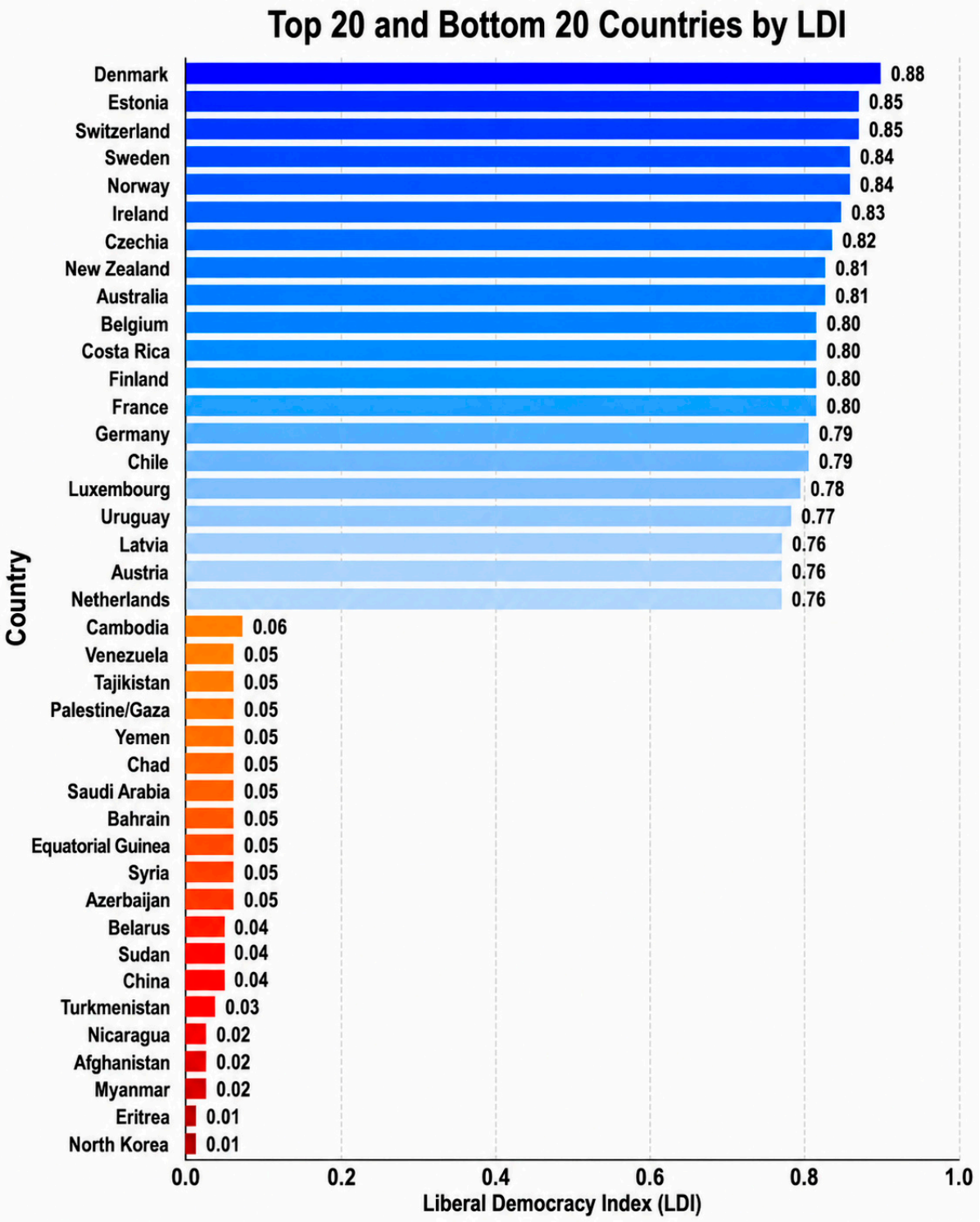

Figure 2. Top 20 and Bottom 20 Countries According to Liberal Democracy Index.

There is a remarkable difference in the extent of democratic development between continents. Figure 4 reveals how Europe is the continent with the highest average values in all aspects of democracy, with extremely high values reported in areas such as LDI = 0.616, in addition to EDI (0.717) and Liberal Component (LCI = 0.801). Oceania is also seen to have equally remarkable values; it tops in individual rights and freedom, especially in the case of the Liberal Component Index (LCI = 0.847).

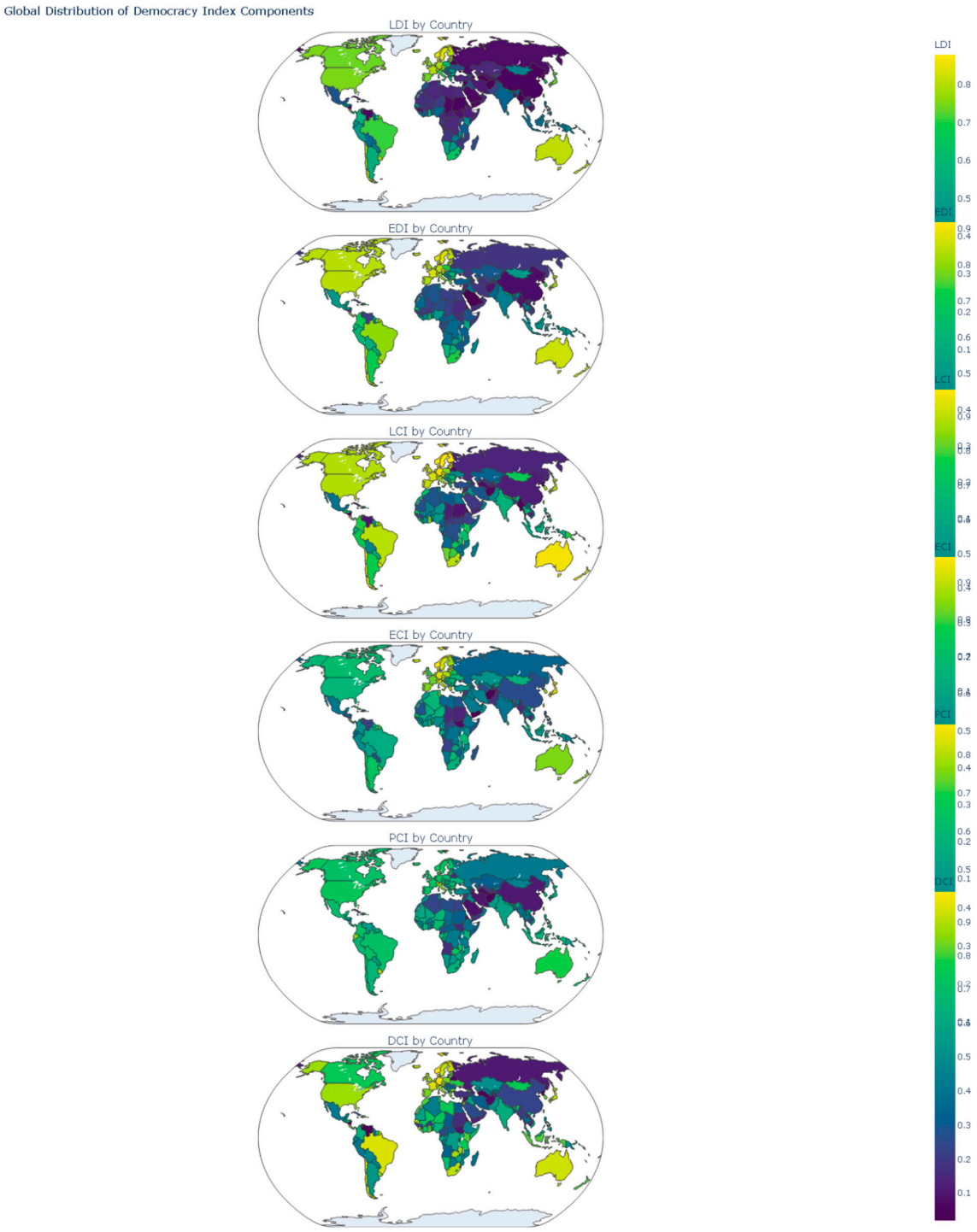

Figure 3. Global Spatial Distribution of Democracy Indicators.

The Americas have the best average in these areas and excel in the fields of EDI (0.617) and LCI (0.637), but at lower levels than in Europe and Oceania. On the other hand, Africa and Asia have the worst averages in the field of democratic indicators. Asia has the worst LDI mean, which amounts to 0.239, and is therefore weak in the fundamental areas of democracy, such as participation and pluralism. The continent of Africa does slightly better in areas related to aspects such as DCI (0.558) and ECI (0.507), but in general has a low democracy aspect. All of the above characteristics clearly illustrate how the political and cultural histories of every continent are recorded in the aspects of democracy.

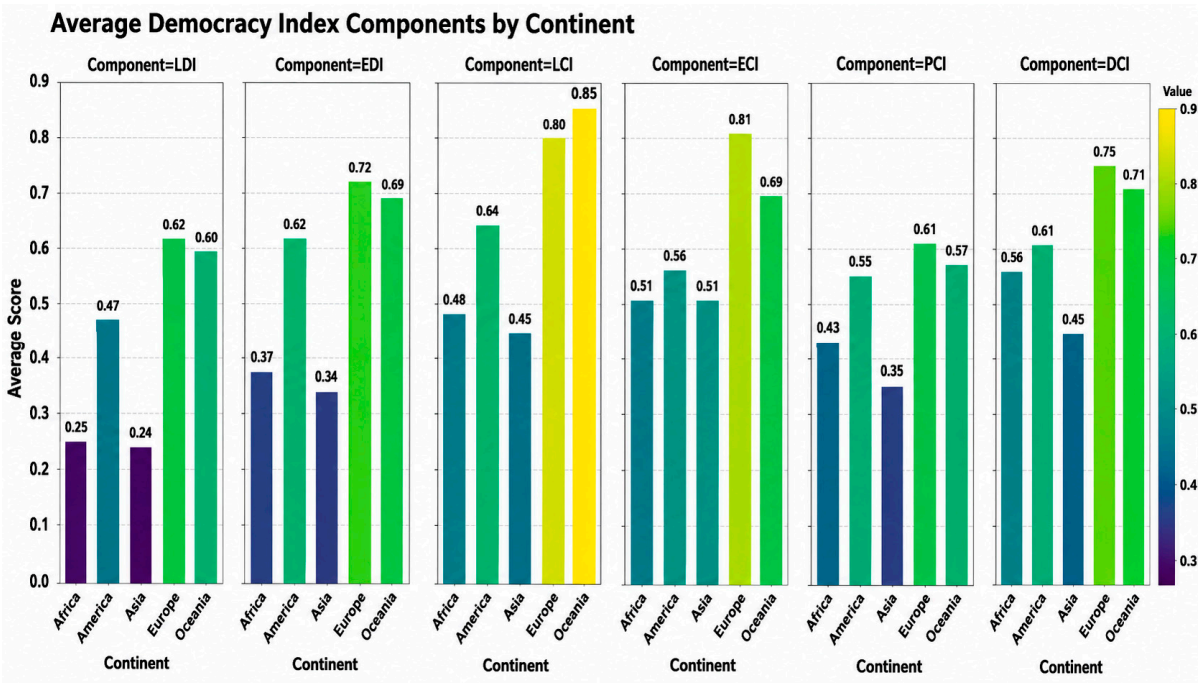

Figure 4. Average Democracy Index Components by Continent.

The present study highlights the results of its regional comparative analysis, shedding light on the distinctions observed at the regional level regarding the concept of democracy. More specifically, based on the V-Dem democracy indexes, the analysis compares the countries, grouping them based on the geographical region they belong to. The results have been included in Figure 5.

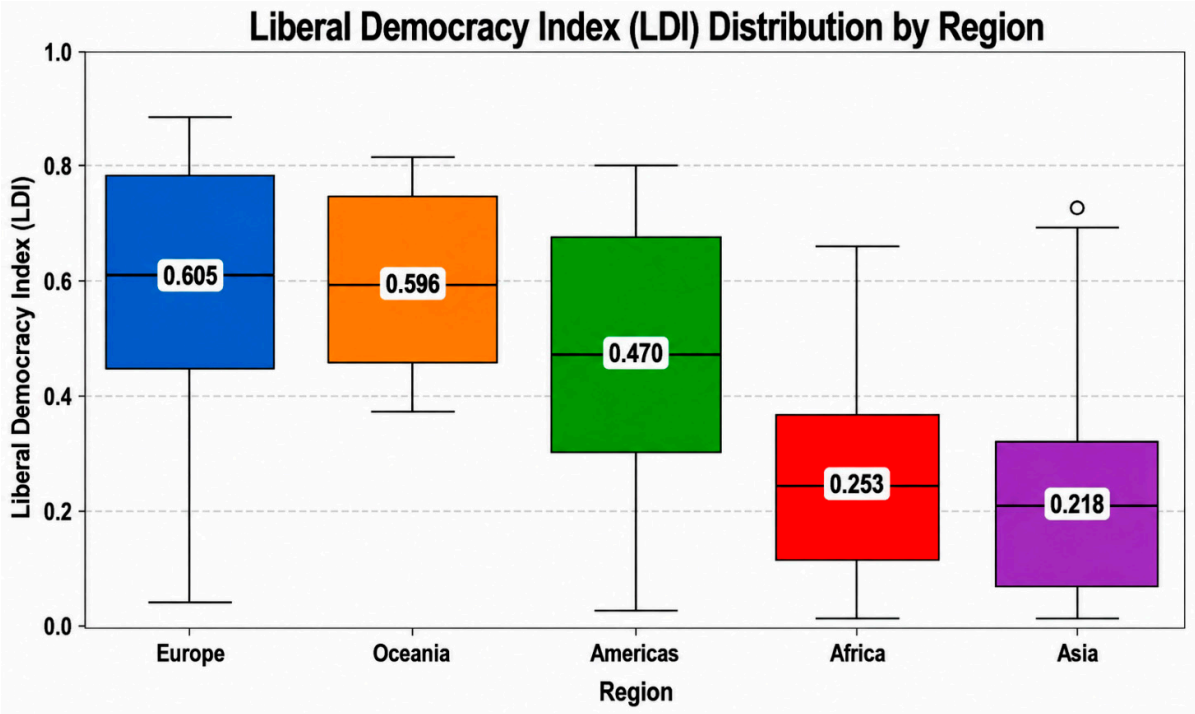

Figure 5. Democracy Index Scores by Region.

From the analysis results presented in Figure 5 below, the Europe and Oceania regions have the highest averages in all the components of democracy. These regions are the ones with the highest averages in the LDI, the EDI, and the LCI. The Americas have a moderate

level of democratic performance compared to the low averages in the other components of democracy in Africa and Asia. This shows that the levels of democracy are differentiated based on geographical locations and are geographically clustered.

In conclusion, overall results confirm that democratic features are not equally represented in all world regions and are connected to regional specific historical, institutional, and socio-political features.

Correlation analysis was applied in the research to graphically represent the direction and strength of the relations among the components that make up the democracy index (LDI, EDI, LCI, ECI, PCI, DCI). The aim is to determine which variables are strongly positively or negatively correlated with each other. Structural analogies can hence be understood, several linear relations can be determined, and an idea is gained about which variables are to be examined together in the analysis. The correlation matrix is presented below in Figure 6.

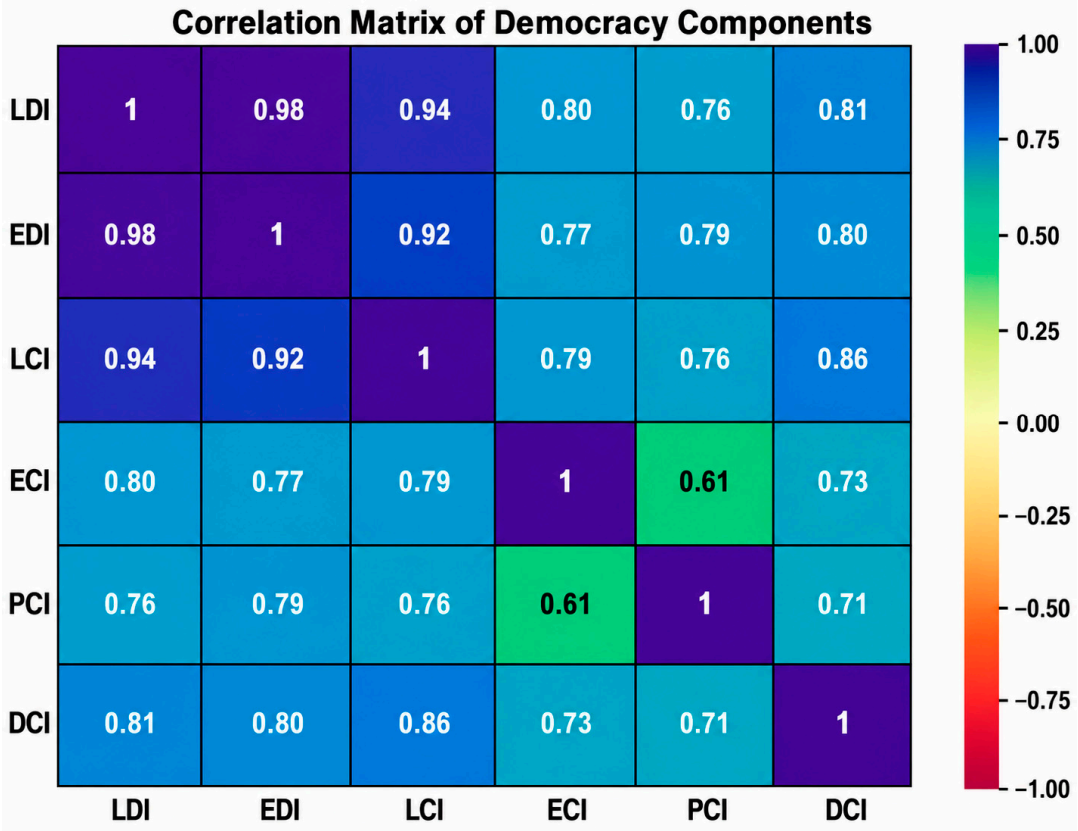

Figure 6. Correlation Matrix of Democracy Indicators.

As Figure 6 reveals, in the correlation matrix between the elements of democracy and their sub-elements, positive and strong relationships are observed between the sub-elements in general. The positive and strong relationship between the LDI and EDI is also observed to be highly correlated ($r = 0.98$), indicating that electoral processes largely establish the measure of the degree of democratization. Similarly, the difference between the LDI and the LCI is also high ($r = 0.94$), indicating that liberal values are strongly related to the level of democracy. The ECI and the PCI correlate with other components at a medium-high level, implying that democratic government and popular participation are complementary dimensions. The DCI, however, is more than 0.7 with other components, and this suggests that discussion and negotiation are extremely crucial in democratic processes. Overall, as seen in the figure, high and positive correlations between the components of democracy support the multidimensional and interactive nature of the democratic system.

4.2. Identification of Global Democracy Regimes Using Unsupervised Learning

The democratic regimes were classified as multidimensional democracy regimes through an unsupervised learning approach based on the following six dimensions of V-Dem: LDI, EDI, LCI, ECI, PCI, and DCI. Contrary to traditional methods, in which a democracy score or a dichotomy of democracy and autocracy is used, the purpose of this paper was to classify countries into clusters of institutional similarity based on dimensions of democracy.

The estimation of the optimal number of clusters was viewed as a problem of model selection rather than searching for a unique optimal solution in statistical terms. Thus, a few different ways to find the optimal number of clusters were suggested, such as the elbow criterion, silhouette coefficient, gap statistic, GMM information criteria, bootstrapping, and consensus clustering.

It can be seen from Figure 7 that the silhouette coefficient has its peak value with two clusters (0.499). Even though the highest possible statistical difference between two major groups has been achieved in this case, a two-cluster solution cannot be enough to capture the institutional differences among contemporary democracies. In other words, there may be nations that possess relatively developed electoral institutions but have less developed liberal, participatory, and/or deliberative elements.

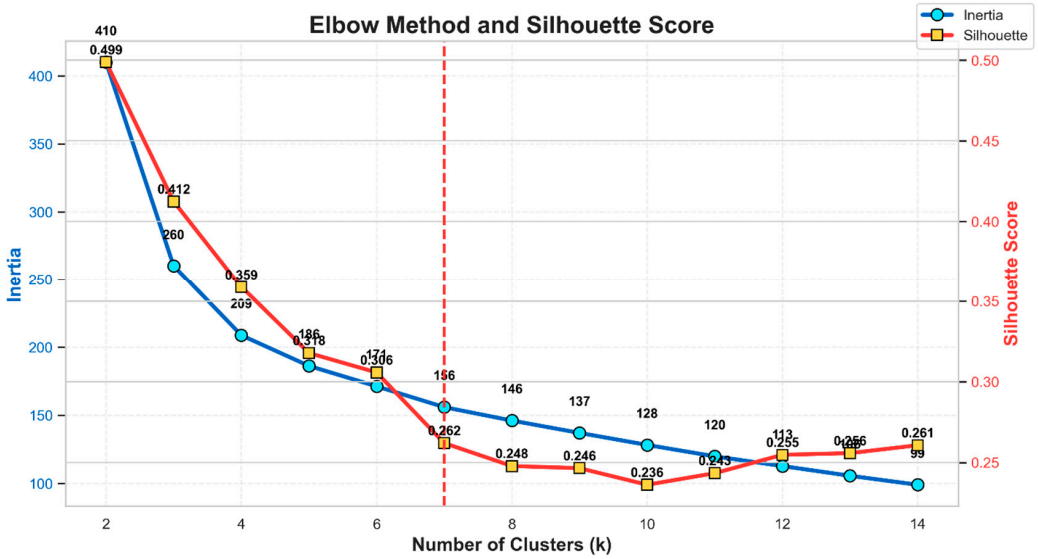

Figure 7. Elbow Method and Silhouette Analysis for Cluster Selection.

Other solutions for clustering were therefore considered. The gap statistic shown in Figure 8 is observed to rise among the other solutions and attain its peak value at ten clusters (gap = 1.447). Nonetheless, because there is no observable structure breakpoint and because more clusters result in finer partitions, the gap statistic is not sufficient on its own.

Additionally, estimation of a GMM was performed as an alternative probabilistic model-based clustering algorithm. From Figure 9, the GMM information criteria do not indicate a consistent optimum: BIC has its minimum value at four clusters (BIC = 919.30), while AIC has its minimum value at ten clusters (AIC = 360.53). The discrepancy also demonstrates that the number of regimes of democracy cannot be decided solely based on a single statistical measure.

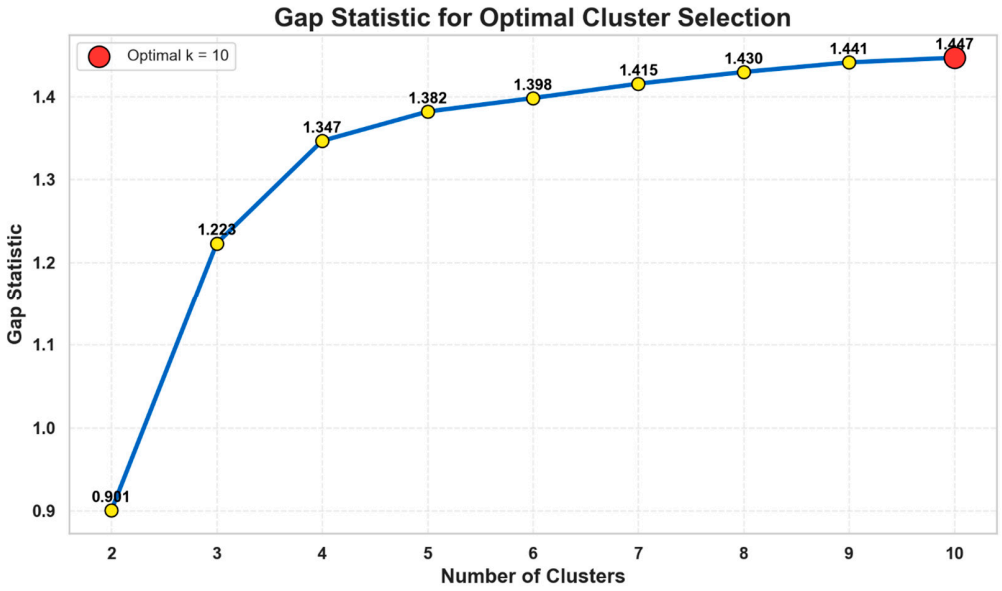

Figure 8. Gap Statistic for Alternative Cluster Solutions.

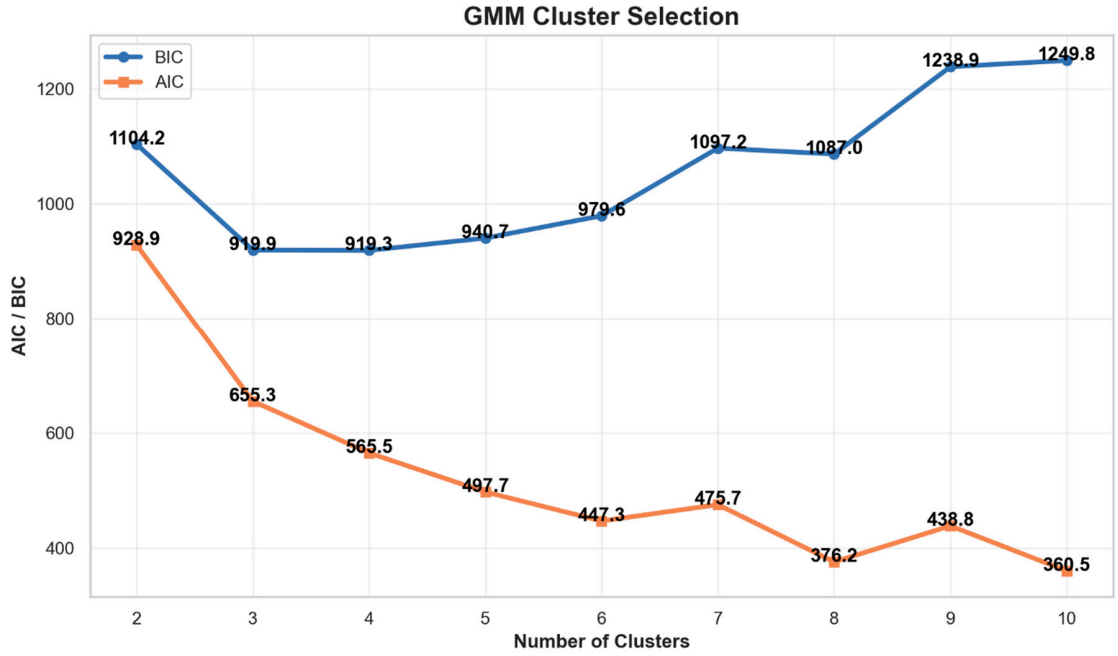

Figure 9. Gaussian Mixture Model Selection Using AIC and BIC.

Given the combined findings, the seven-cluster K-means approach was kept as the primary classification of the democratic regime type. This decision is to be regarded as a scientifically meaningful and interpretable classification and does not serve as proof that seven is the only optimal number of regimes. The elbow, Silhouette, gap statistic, and GMM information criterion all gave additional but diverging evidence about the number of clusters to be considered. Moreover, the use of bootstrap stability analysis and consensus clustering allowed us to assess the stability of the chosen classification, and the comparison with GMM made it possible to evaluate the sensitivity of cluster assignments to the clustering algorithm used. However, it still cannot be argued whether the interpretation of the regimes would change with any other value of K. Therefore, the seven-cluster classification is the primary one.

4.3. Assessment of Cluster Existence and Robustness

The stability and reliability of the clusters to be interpreted were tested before proceeding to their interpretation through various validation techniques. Initially, the Hopkins statistics were computed to test for the presence of a cluster tendency in the six-dimensional democracy space. The Hopkins value (0.766) shows a high cluster tendency, meaning that the institution types found are not randomly distributed but have certain identifiable patterns.

In addition, cluster stability was computed using bootstrap analysis, which involved 1000 replications. The Jaccard similarity coefficient was applied to calculate the cluster stability based on the comparison of initial clustering results and the clustering results obtained through bootstrap analysis. The stability of clusters depended on the type of cluster, as demonstrated in Figure 10 below. The highest values of stability were exhibited by Cluster 2 and Cluster 6, which were 0.761 and 0.750, respectively. Moderate stability levels for the other clusters were as follows: Cluster 1 (0.587), Cluster 5 (0.585), Cluster 3 (0.559), Cluster 4 (0.532), and Cluster 0 (0.466).

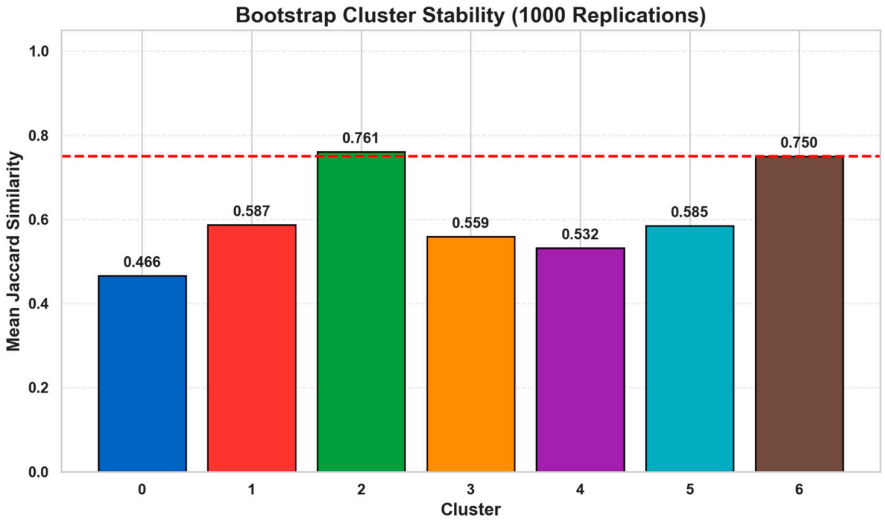

Figure 10. Bootstrap Jaccard Stability of Democracy Regimes (1000 Replications).

Third, the process of consensus clustering was conducted to see if there would be any stability in the country classifications under repeated bootstrap estimates. The consensus matrix was constructed through 1000 bootstrap replications, followed by hierarchical clustering based on the similarity structure obtained. As can be seen from Figure 11, there are evident block structures of countries in the consensus matrix.

Further examination of the correspondence between the initial K-means classification and the result of consensus clustering was conducted through ARI and NMI. As shown in Table 1, the evaluation yielded an excellent match (ARI = 0.7637; NMI = 0.8801), indicating that the democracy regimes are highly consistent across different bootstrapping samples.

Table 1. Robustness Validation Results of the Seven-Cluster Democracy Typology.

Validation Procedure	Comparison	Metric	Result
Cluster tendency assessment	Democracy feature space	Hopkins statistic	0.766
Bootstrap stability	Original K-means vs. bootstrap solutions	Jaccard similarity	0.466–0.761
Consensus clustering	K-means vs. consensus solution	ARI	0.7637
Consensus clustering	K-means vs. consensus solution	NMI	0.8801

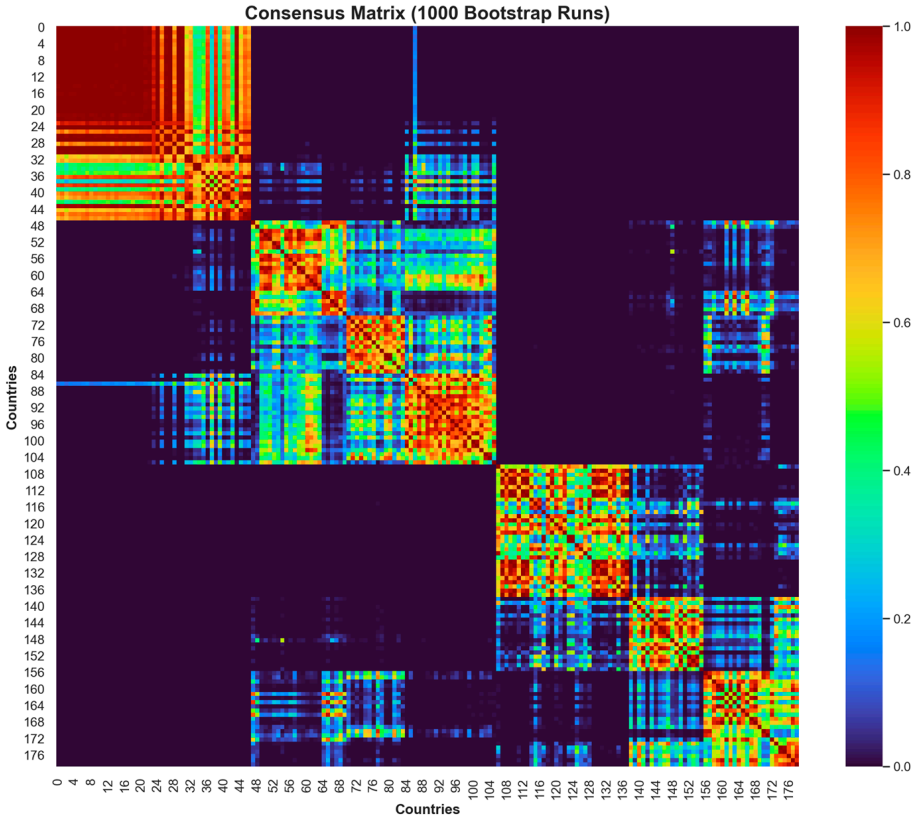

Figure 11. Consensus Matrix Based on 1000 Bootstrap Replications.

Taken together, the results of the validation procedure show the presence of a non-random clustering pattern and an appreciable level of reproducibility in the case of the seven-cluster classification. The concordance between the initial K-means and the consensus classifications (ARI = 0.7637; NMI = 0.8801) indicates the general agreement of the classification, while the Jaccard coefficients of the bootstrapped regimes vary within a range of 0.466–0.761. These results confirm the practical value of the seven-cluster classification but fail to prove the unique optimality of the classification or invariance of the substantive conclusions to other numbers of clusters.

4.4. Comparison of K-Means and Gaussian Mixture Model Classifications

The sensitivity of the seven-cluster primary K-means cluster typology to the method of clustering was tested by comparing it with the probabilistic GMM. The use of GMM was not meant to replace the primary K-means clustering but was rather an added robustness test using a different approach to clustering. In contrast to the K-means, which finds clusters through the minimization of Euclidean distances within the clusters, GMM finds the hidden regimes based on covariance structure.

The comparison of these two classification systems is performed using the ARI and NMI. The agreement and correspondence of the K-means and GMM classifications are displayed in the cluster matching table in Figure 12.

Despite that, there is still a relatively strong level of similarity between these methods, given that ARI = 0.5275 and NMI = 0.6605. Such results imply that these metrics reveal a certain amount of similarity but are far from full. This can be considered natural, since K-means and GMM are characterized by different assumptions about clusters and their properties.

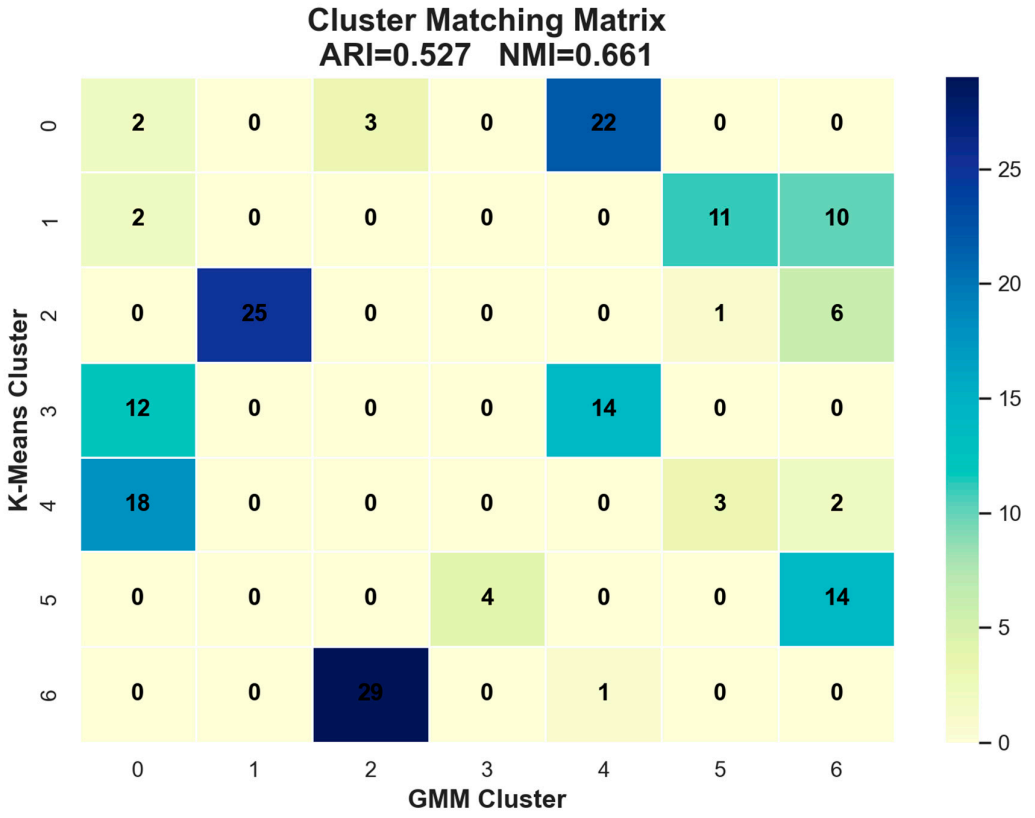

Figure 12. Cluster Matching Matrix Between K-Means and Gaussian Mixture Model Classifications.

As can be seen from the cluster matching matrix (Figure 12), there are some common major democracy configurations, but at the same time, some differences can be observed. They occur mostly at the intermediate stages, since the classification of countries located between democratic systems and hybrid regimes is more complicated due to peculiarities of their institutions.

Thus, the degree of congruence observed between the K-means and GMM techniques should not be construed as inconsistent methodology. Rather, it underscores the complex and continuous character of democracy around the world, with certain states having transitional statuses in the multidimensional institutional universe. It appears that the identified democracy types can be discerned. However, their boundaries are not exclusively distinct, especially in the cases of hybrids and intermediaries.

Therefore, the results from the GMM approach should be considered a complementary robustness check and not confirmation of a similar classification. In other words, the moderately high correspondence between the two approaches (ARI = 0.5275; NMI = 0.6605) suggests that there are some common institutional arrangements across the two approaches, yet country assignment depends heavily on the specific assumptions embedded in the chosen method of clustering. This comparison shows that the membership in regimes varies depending on the method of analysis. However, it does not shed light on the robustness of the main findings with respect to different numbers of clusters. Thus, the classification into seven regimes stays as a main one.

4.5. Democracy Regime Profiles and Global Distribution

The main classification by K-means clustered seven democracy regimes, whose institutional features are described in Table 2 below. The seven clusters were chosen based on the quality, stability, and interpretability of the clusters obtained. Estimation by means of a

Gaussian Mixture Model (GMM), as discussed in Section 4.4, was used as a robustness test and did not change regimes membership, as seen in Tables 2 and 3.

Table 2. Democracy Regime Profiles Based on the Primary K-means Classification.

Regime	Democracy Regime	LDI	EDIL	IEC	ECI	PCI	DCI	Countries (N)
0	Low Institutional Democracy Regime	0.060	0.157	0.161	0.281	0.205	0.189	32
1	Restricted Institutional Regime	0.122	0.227	0.321	0.562	0.303	0.292	18
2	Emerging Electoral Regime	0.184	0.310	0.422	0.438	0.474	0.567	23
3	Hybrid Institutional Regime	0.324	0.444	0.649	0.668	0.489	0.742	23
4	Established Electoral Democracy Regime	0.448	0.605	0.698	0.555	0.572	0.550	26
5	Advanced Liberal Democracy Regime	0.601	0.720	0.827	0.754	0.564	0.806	27
6	Consolidated Liberal Democracy Regime	0.779	0.848	0.930	0.870	0.664	0.895	30

The regimes have been defined according to means on six dimensions provided by V-Dem: Liberal Democracy Index (LDI), Electoral Democracy Index (EDI), Liberal Component Index (LCI), Egalitarian Component Index (ECI), Participatory Component Index (PCI), and Deliberative Component Index (DCI). Means for these dimensions and number of countries in each regime are described in Table 2 below.

From the results obtained, it is clear that there is a lot of variation in terms of institutional heterogeneity among the seven regimes. The Consolidated Liberal Democracy Regime ranks topmost in terms of means for LDI (0.779), EDI (0.848), LCI (0.930), ECI (0.870), and DCI (0.895). This is due to the high scores in all institutional dimensions. The Advanced Liberal Democracy Regime also has an institutional profile which scores high in LDI (0.601), EDI (0.720), LCI (0.827), ECI (0.754), and DCI (0.806). Nevertheless, the mean score of its PCI is lower than that of the Emerging Electoral and Hybrid Institutional Regimes. The Established Electoral Democracy Regime falls at the medium-high position, with its EDI (0.605) being higher than LDI (0.448). The Established Electoral Democracy Regime occupies an intermediate-to-high position, with its EDI (0.605) exceeding its LDI (0.448), while its remaining dimension scores range from 0.550 to 0.698.

The Emerging Electoral Regime is characterized by comparably low LDI (0.184) and EDI (0.310) means and a comparatively high DCI mean (0.567). The Hybrid Institutional Regime has an inverse combination of indices, meaning that its LCI (0.649), ECI (0.668), and DCI (0.742) means are relatively high compared to its LDI (0.324) and EDI (0.444) means. This shows that the regimes have different mixes of institutional characteristics and thus they cannot be differentiated based on the same progression along the dimensions.

On the lower end of the classification, the Restricted Institutional Regime has relatively low means in all six dimensions, with the lowest mean being LDI (0.122) while the highest mean is ECI (0.562). The Low Institutional Democracy Regime has the lowest LDI (0.060), EDI (0.157), and LCI (0.161) means, among others. All together, these profiles demonstrate the diversity of institutional mixtures captured by the basic K-means clustering.

Table 3. Institutional Entropy and Complexity Index across Democracy Regimes.

Regime	Democracy Regime	Countries (N)	Country Membership	Average Entropy	Complexity Index
0	Low Institutional Democracy Regime	32	Afghanistan, Angola, Azerbaijan, Bahrain, Bangladesh, Burundi, CAR, Cambodia, Chad, China, Egypt, El Salvador, Equatorial Guinea, Eritrea, Eswatini, Haiti, Iran, Myanmar, Nicaragua, North Korea, Palestine/Gaza, Qatar, Russia, Saudi Arabia, South Sudan, Sudan, Syria, Tajikistan, Turkmenistan, UAE, Venezuela, Yemen	0.3223	0.5400
1	Restricted Institutional Regime	18	Algeria, Belarus, Burkina Faso, Cameroon, Cuba, Djibouti, Ethiopia, Guinea, Guinea-Bissau, Hong Kong, Kazakhstan, Kuwait, Kyrgyzstan, Laos, Oman, Palestine/West Bank, Türkiye, Uzbekistan	0.4794	0.8034
2	Emerging Electoral Regime	23	Comoros, Congo, DRC, Gabon, India, Iraq, Lebanon, Libya, Madagascar, Mali, Mauritania, Mexico, Mozambique, Pakistan, Philippines, Rwanda, Somalia, Somaliland, Thailand, Togo, Uganda, Vietnam, Zimbabwe	0.4472	0.7495
3	Hybrid Institutional Regime	23	Armenia, Benin, BiH, Fiji, Georgia, Indonesia, Ivory Coast, Jordan, Liberia, Malaysia, Maldives, Mauritius, Mongolia, Morocco, Niger, Serbia, Sierra Leone, Singapore, Tanzania, Tunisia, Ukraine, Zambia, Zanzibar	0.5689	0.9533
4	Established Electoral Democracy Regime	26	Albania, Argentina, Bolivia, Botswana, Colombia, Dominican Republic, Ecuador, Guatemala, Guyana, Honduras, Hungary, Kenya, Kosovo, Malawi, Namibia, Nepal, Nigeria, North Macedonia, Papua New Guinea, Paraguay, Peru, Romania, Slovakia, Solomon Islands, Sri Lanka, Timor-Leste	0.5967	1.0000
5	Advanced Liberal Democracy Regime	27	Barbados, Bhutan, Brazil, Bulgaria, Canada, Cape Verde, Croatia, Cyprus, Gambia, Ghana, Greece, Israel, Jamaica, Lesotho, Malta, Moldova, Montenegro, Panama, Poland, S.Tomé & P., Senegal, Seychelles, South Africa, South Korea, Suriname, Trinidad and Tobago, Vanuatu	0.3576	0.5992
6	Consolidated Liberal Democracy Regime	30	Australia, Austria, Belgium, Chile, Costa Rica, Czechia, Denmark, Estonia, Finland, France, Germany, Iceland, Ireland, Italy, Japan, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovenia, Spain, Sweden, Switzerland, Taiwan, USA, United Kingdom, Uruguay	0.1266	0.2121

The country assignments from the first phase of K-means classification remain the same in the second entropy analysis stage. Therefore, Table 2 shows the regime structure which remains constant throughout Section 4.6. Figure 13 shows the geographical distribution of the seven regimes.

The geographical pattern indicates that the institutional configurations identified do not relate to a specific geographic pattern but instead exist in different geographical locations. The geographical pattern is in keeping with the multidimensional aspect of the classifications where similar institutional configurations can be seen in different geographic regions.

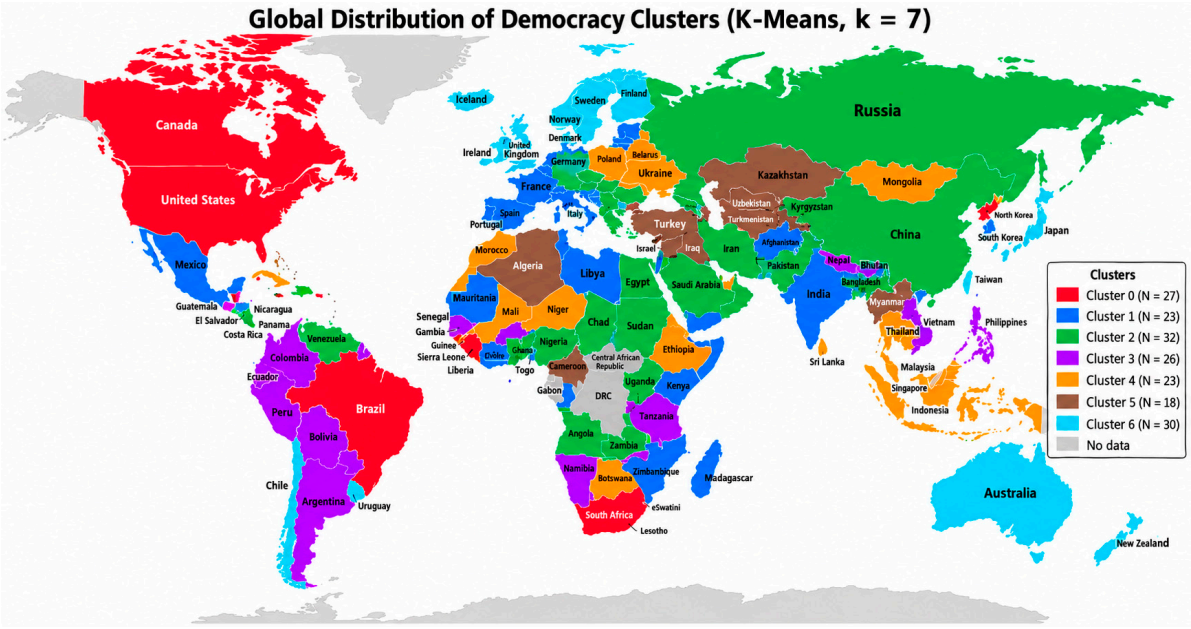

Figure 13. Global Spatial Distribution of Democracy Regimes.

4.6. Institutional Complexity and Regime Heterogeneity

Entropy is applied as an auxiliary measure of institutional diversity in regimes as opposed to being a supplementary classifying technique. The seven regime affiliations are derived from the primary K-means classifying algorithm described in Section 4.5.

For each regime, the six V-Dem indicators are classed into three institutional states, low, medium, and high, utilizing cutoff points of 1/3 and 2/3. The Shannon entropy is then computed for each indicator with reference to the actual frequency of the three institutional states and correcting for sample size with the Miller–Madow correction. The average of the six entropies provides the average entropy for each regime [15,16].

The cutoffs of 1/3 and 2/3 create equal intervals of the democracy variables in terms of the normalized democracy indicator and thus define three institutional states. The values of entropy and the ordering of the regimes, however, may be affected by the chosen cutoffs since discretization influences the allocation of observations to various states. Thus, the entropy and Complexity Index numbers below must be understood in the context of these cutoffs. Without any comparison to other sets of cutoffs, there is no evidence that the calculated institutional heterogeneity levels are independent of the cutoff choices made.

Table 3 shows the entropy and Complexity Index numbers derived from the calculations. Country regimes in this table are defined based on the primary K-means classification described above in Table 2. The entropy does not produce any classification. It describes the distribution of institutional states in each predefined regime.

As evident from Table 3, there are considerable variations in within-regime institutional heterogeneity. For instance, the Established Electoral Democracy Regime has the largest average entropy value (0.5967) which implies that it has the highest Complexity Index (1.0000), while the Hybrid Institutional Regime comes second (0.5689; 0.9533). On the other hand, the Restricted Institutional Regime and Emerging Electoral Regime exhibit moderate heterogeneity levels, while the Advanced Liberal and Low Institutional Regimes display smaller entropy values. Lastly, the Consolidated Liberal Democracy Regime has the smallest average entropy value (0.1266) and Complexity Index (0.2121).

Generally, the entropy-based measure is supplementary to the main K-means categorization, as it measures within-regime heterogeneity without affecting regime classification.

On the other hand, the GMM analysis is independent and serves as a robustness confirmation for the main classification.

4.7. External Validation of Democracy Regimes Using the World Justice Project Rule of Law Index

External validity of the K-means democracy regime typology was evaluated by using the World Justice Project (WJP) Rule of Law Index. Measures from the WJP were not used in the construction of the seven democracy regimes and thus were kept exclusively for external validation. This test involves checking for systematic differences between the regimes that were formed based on the V-Dem indicators within another independent institutional dimension, the rule of law.

External validation was done by aligning the K-means country cluster memberships to those of the WJP dataset. The WJP-aligned sample includes 119 countries and is a subset of the 179 countries of the V-Dem categorization. The regime memberships were not recomputed using the WJP dataset. In other words, each aligned country has retained its regime membership based on the K-means classification. This implies that the WJP sample size will be equal to, or less than, but never greater than, the full sample regime size. The consistency check for each regime has shown that the WJP-aligned regime membership is smaller than or equal to the full sample regimes' sizes: 10, 10, 16, 17, 22, 18, and 26 countries for Regimes 0 to 6, respectively.

As can be seen from Table 4 below, there are considerable disparities between the average WJP Rule of Law scores for the different regimes. For instance, the Low Institutional Democracy Regime (Regime 0) has an average WJP score of 0.371 (SD = 0.055, N = 10), whereas the Restricted Institutional Regime (Regime 1) has an average score of 0.457 (SD = 0.071, N = 10). The Emerging Electoral Regime (Regime 2) has an average WJP score of 0.439 (SD = 0.068, N = 16), while the Hybrid Institutional Regime (Regime 3) has an average score of 0.518 (SD = 0.087, N = 17). The Established Electoral Democracy Regime (Regime 4) has an average WJP score of 0.498 (SD = 0.064, N = 22), whereas the Advanced Liberal Democracy Regime (Regime 5) has an average score of 0.588 (SD = 0.079, N = 18). The Consolidated Liberal Democracy Regime (Regime 6) has the highest average WJP score of 0.776 (SD = 0.071, N = 26).

Table 4. Regime-Level Descriptive Statistics for the WJP Rule of Law Index.

Democracy Regime	Regime	N	Mean WJ	SD	Minimum	Maximum
Low Institutional Democracy Regime	0	10	0.371	0.055	0.310	0.470
Restricted Institutional Regime	1	10	0.457	0.071	0.360	0.580
Emerging Electoral Regime	2	16	0.439	0.068	0.360	0.630
Hybrid Institutional Regime	3	17	0.518	0.087	0.430	0.780
Established Electoral Democracy Regime	4	22	0.498	0.064	0.370	0.620
Advanced Liberal Democracy Regime	5	18	0.588	0.079	0.490	0.800
Consolidated Liberal Democracy Regime	6	26	0.776	0.071	0.660	0.900
Total	—	119	—	—	—	—

The differences between WJP scores were tested by means of one-way ANOVA. As can be seen from Table 5, the results of ANOVA indicate significant differences between WJP scores in terms of seven regimes, $F(6, 112) = 65.385$, $p < 0.001$. The value of eta squared ($\eta^2 = 0.778$) means that 77.8% of the total variance in WJP scores can be explained by the differences between regimes. The same pattern can be found in the results of the non-parametric Kruskal–Wallis test, $H(6) = 85.889$, $p < 0.001$.

Table 5. ANOVA and Non-Parametric Validation of Democracy Regimes Using the WJP Index.

Analysis Result	Value
Matched sample size	N = 119
Number of regimes	7
Between-group df	6
Within-group df	112
ANOVA F	65.385
p-value	<0.001
Eta-squared (η^2)	0.778
Kruskal–Wallis H(6)	85.889
Kruskal–Wallis *p*	<0.001

A post-hoc Tukey HSD test was carried out to establish any pairwise differences in mean WJP scores. Significant differences were found in 13 of the 21 possible pairwise comparisons. Regime 6 showed significant differences from all other regimes ($p < 0.001$ for all adjusted values). Regime 5 showed significant differences from Regimes 0 ($p < 0.001$), 1 ($p = 0.0002$), 2 ($p < 0.001$), and 4 ($p = 0.0025$); however, it did not show significant differences from Regime 3 ($p = 0.0653$). Regime 0 was significantly different from Regimes 3 ($p < 0.001$), 4 ($p = 0.0002$), and 5 ($p < 0.001$). Other significant differences were seen between Regimes 2–3 ($p = 0.0362$), 2–5 ($p < 0.001$), 2–6 ($p < 0.001$), 3–6 ($p < 0.001$), 4–5 ($p = 0.0025$), and 4–6 ($p < 0.001$). These results are presented in Table 6.

The Kruskal–Wallis test was used to assess the robustness of ANOVA results in a non-parametric manner. It also shows significant differences among the seven regimes, $H(6) = 85.889$, $p < 0.001$. Apart from the general WJP index, analyses of the eight WJP subindices showed statistically significant differences among the regimes. The eta-squared varies between 0.367 in the case of Order and Security and 0.843 in the case of Fundamental Rights. The other indices also show substantial variation across regimes, with η^2 equaling 0.793 for Constraints on Government Powers, 0.682 for Absence of Corruption, 0.783 for Open Government, 0.715 for Regulatory Enforcement, 0.688 for Civil Justice, and 0.686 for Criminal Justice.

Generally, the findings from WJP add independent, external evidence to suggest differences among the seven K-means primary regimes. In fact, since the indicators used in WJP are not among those used to create the primary regimes, the differences between the regimes observed can be considered external validation instead of internal clustering of the regimes. Consistency in the results obtained from ANOVA and Kruskal–Wallis tests coupled with the differences revealed through Tukey HSD analysis suggests that regime categorization is significantly different in terms of the external rule of law indicators.

Table 6. Tukey HSD Post-Hoc Comparisons of WJP Scores.

Regime 1	Regime 2	Mean Difference	Adjusted p	95% CI	Significant
Regime 0	Regime 1	0.0860	0.1158	[−0.0107, 0.1827]	No
Regime 0	Regime 2	0.0684	0.2280	[−0.0188, 0.1556]	No
Regime 0	Regime 3	0.1466	<0.001	[0.0604, 0.2328]	Yes
Regime 0	Regime 4	0.1267	0.0002	[0.0442, 0.2092]	Yes
Regime 0	Regime 5	0.2173	<0.001	[0.1320, 0.3026]	Yes
Regime 0	Regime 6	0.4052	<0.001	[0.3247, 0.4856]	Yes
Regime 1	Regime 2	−0.0176	0.9965	[−0.1048, 0.0696]	No
Regime 1	Regime 3	0.0606	0.3526	[−0.0256, 0.1468]	No
Regime 1	Regime 4	0.0407	0.7546	[−0.0418, 0.1232]	No
Regime 1	Regime 5	0.1313	0.0002	[0.0460, 0.2166]	Yes
Regime 1	Regime 6	0.3192	<0.001	[0.2387, 0.3996]	Yes
Regime 2	Regime 3	0.0783	0.0362	[0.0029, 0.1536]	Yes
Regime 2	Regime 4	0.0584	0.1821	[−0.0127, 0.1294]	No
Regime 2	Regime 5	0.1490	<0.001	[0.0746, 0.2233]	Yes
Regime 2	Regime 6	0.3368	<0.001	[0.2681, 0.4055]	Yes
Regime 3	Regime 4	−0.0199	0.9781	[−0.0898, 0.0499]	No
Regime 3	Regime 5	0.0707	0.0653	[−0.0025, 0.1438]	No
Regime 3	Regime 6	0.2585	<0.001	[0.1910, 0.3260]	Yes
Regime 4	Regime 5	0.0906	0.0025	[0.0219, 0.1593]	Yes
Regime 4	Regime 6	0.2784	<0.001	[0.2158, 0.3411]	Yes
Regime 5	Regime 6	0.1878	<0.001	[0.1215, 0.2541]	Yes

4.8. Explainable Machine Learning Analysis of Predictive Contributions Among Democracy Dimensions

After defining the global democracy regime and the analysis of institutional complexity, an XAI methodology was developed to analyze the relative contributions of democratic dimensions to predict the LDI. We do not intend to identify causality between democracy indicators, but we do seek to determine the predictive contributions of institutional democracy dimensions from the V-Dem liberal democracy approach.

Five democratic dimensions, such as EDI, LCI, ECI, PCI, and DCI, were taken as explanatory variables, whereas LDI was treated as the target variable. Nonlinear machine learning methods were applied and compared through R^2, RMSE, and MAE performance criteria.

The comparative predictive performance of evaluated machine learning models is shown in Figure 14 below. The outcomes show that nonlinear ensemble models have better predictive ability than others in terms of estimating the LDI framework.

As can be seen in Figure 14, nonlinear algorithms based on ensembles gave the best prediction performance. The Extra Trees algorithm showed the best performance for all models considered, with an R^2 score of 0.9991, RMSE score of 0.0285, and MAE score of 0.0213. Moreover, the Gradient Boosting, CatBoost, and Nu-SVR algorithms gave highly accurate predictions, with R^2 scores higher than 0.997. It can be said that nonlinear learning algorithms managed to identify the complicated relations between democratic dimensions.

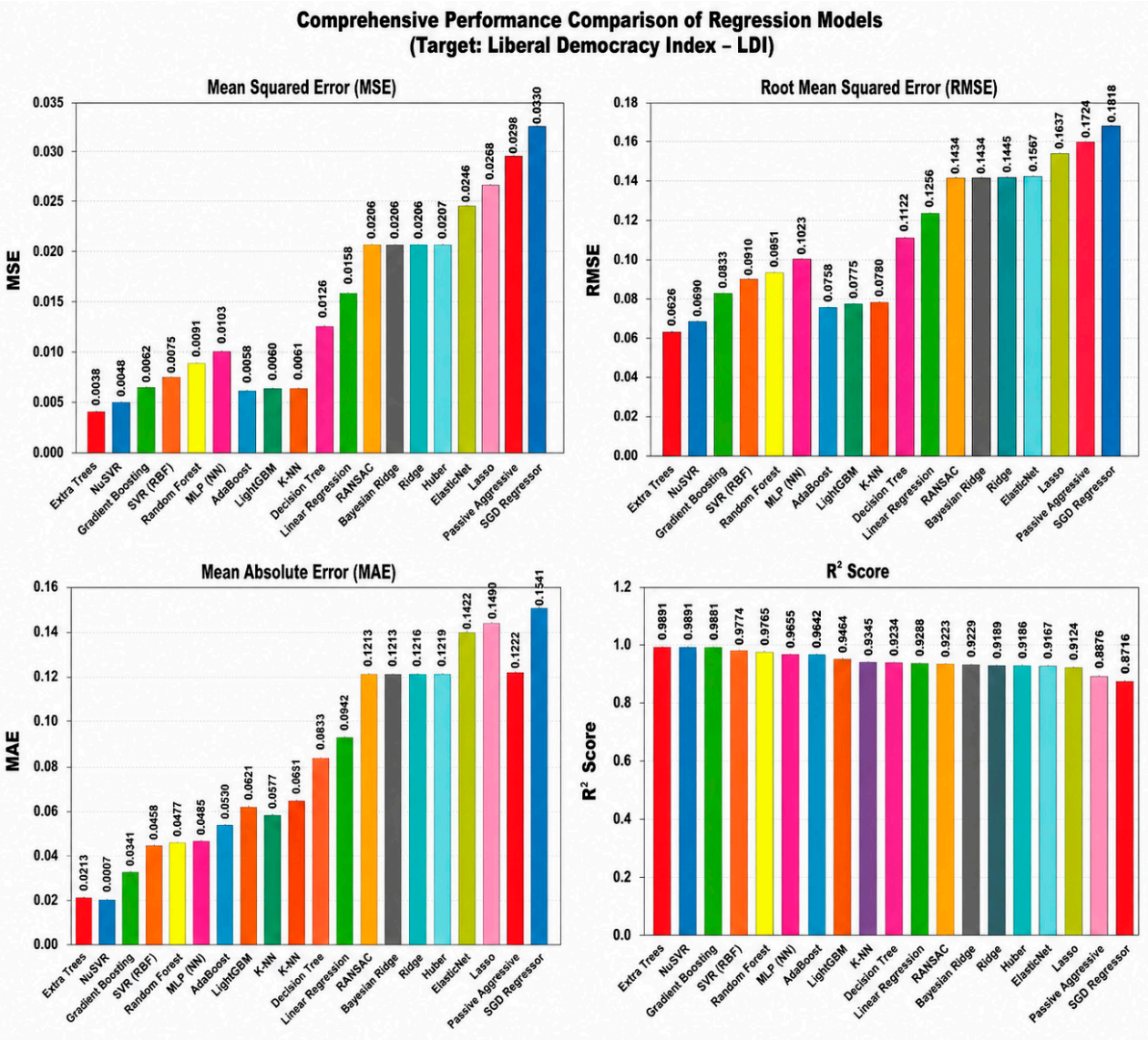

Figure 14. Predictive Performance of Machine Learning Models for Liberal Democracy Index Estimation.

The contribution of each democratic dimension was evaluated using a global feature importance method based on SHAP for the best performing model. The feature importance values generated using the SHAP method are shown in Figure 15.

The SHAP findings shown in Figure 15 show that EDI and LCI have the greatest contribution to predicting LDI. The mean SHAP absolute values were 0.5092 for EDI and 0.3022 for LCI, while those of DCI (0.0306), ECI (0.0226), and PCI (0.0141) were relatively low. This shows that there is more prediction power in the electoral competitiveness and liberal institution constraint features of V-Dem liberal democracy.

A scenario analysis of feature effects based on a ±1 standard deviation using SHAP was conducted. Figure 16 shows the estimated prediction changes of LDI values in both cases.

As depicted in Figure 16, an increase in EDI by one standard deviation resulted in a prediction gain for the LDI of 0.5121, while a decrease in EDI resulted in a negative effect on the prediction of −0.4724. Likewise, LCI had effects of +0.3174 and −0.3806, respectively. By contrast, DCI, ECI, and PCI had rather small effects of +0.0256/−0.0346, +0.0213/−0.0214, and +0.0131/−0.0161, respectively.

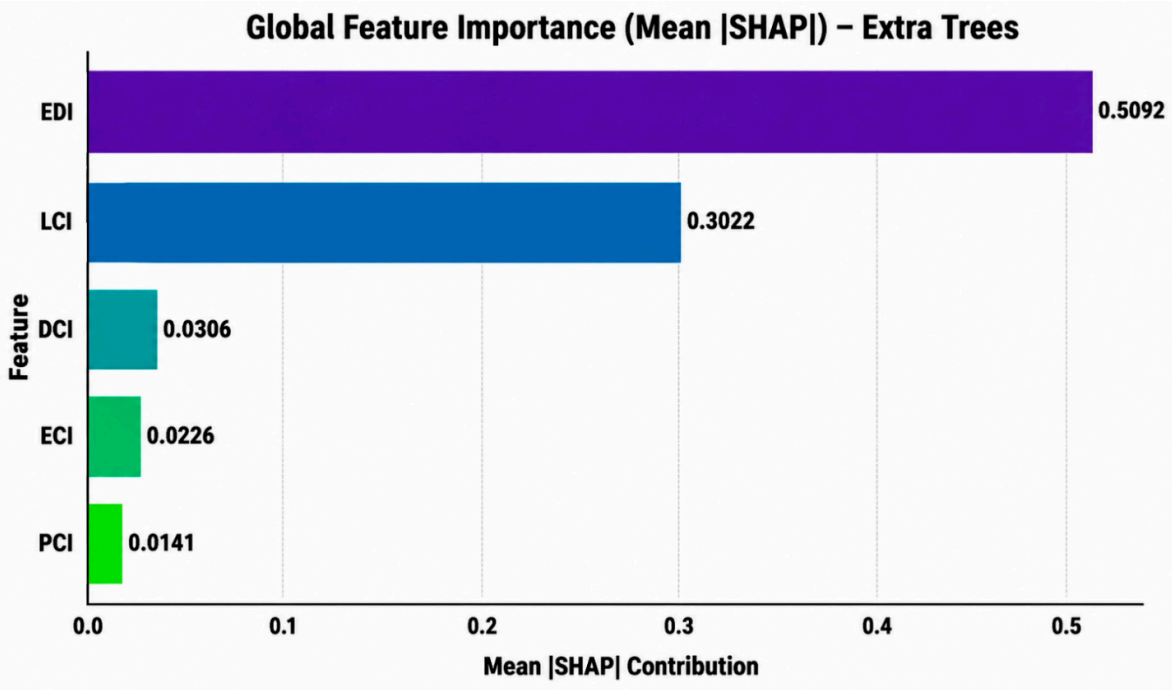

Figure 15. Global Feature Importance Based on Mean Absolute SHAP Values.

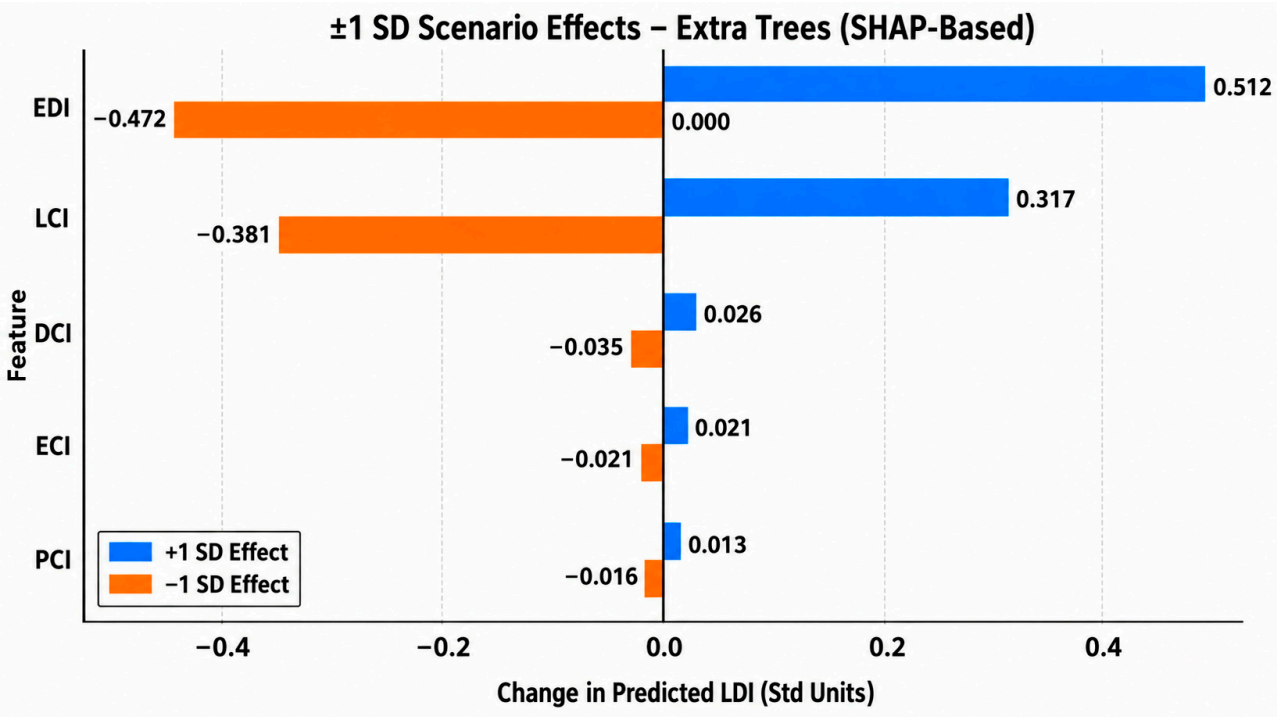

Figure 16. SHAP-Based ±1 Standard Deviation Scenario Effects on LDI Prediction.

The robustness of the SHAP-based results was tested using local interpretability approach based on the LIME explanation. Local feature importance values are shown in Figure 17.

The LIME results presented in Figure 17 confirm the major contributions of EDI (0.6277) and LCI (0.3958). However, the contributions of DCI (0.0293), ECI (0.0188) and PCI (0.0185) were still significantly lower. The consistency between the SHAP and LIME anal-

yses demonstrates that the established contribution trends are not related to a particular explanation method.

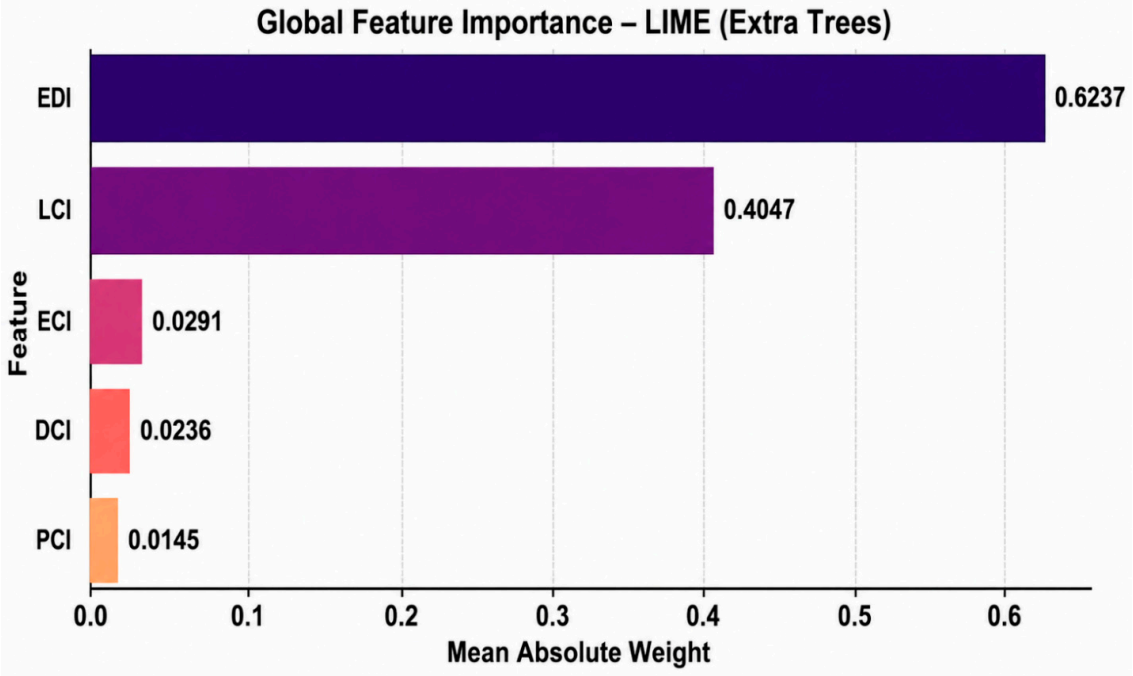

Figure 17. Local Feature Importance Based on LIME Explanations.

The comparison of SHAP and LIME explanations is provided in Figure 18.

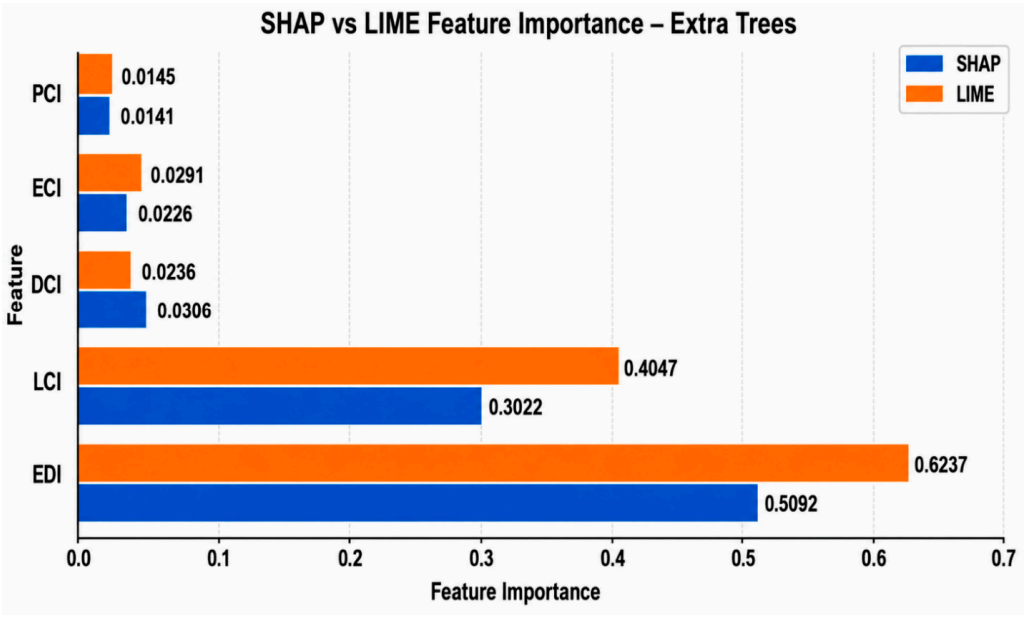

Figure 18. Comparison of SHAP and LIME Feature Importance.

It is clear from Figure 18 that the SHAP and LIME methods consistently detect EDI and LCI as two most important democratic dimensions for predictions of LDI. Other dimensions have less significant contributions to predictive performance.

Finally, Permutation Feature Importance was evaluated as another way to test the robustness of the model. Figure 19 shows the influence of random shuffling of each democratic dimension on the performance of the model.

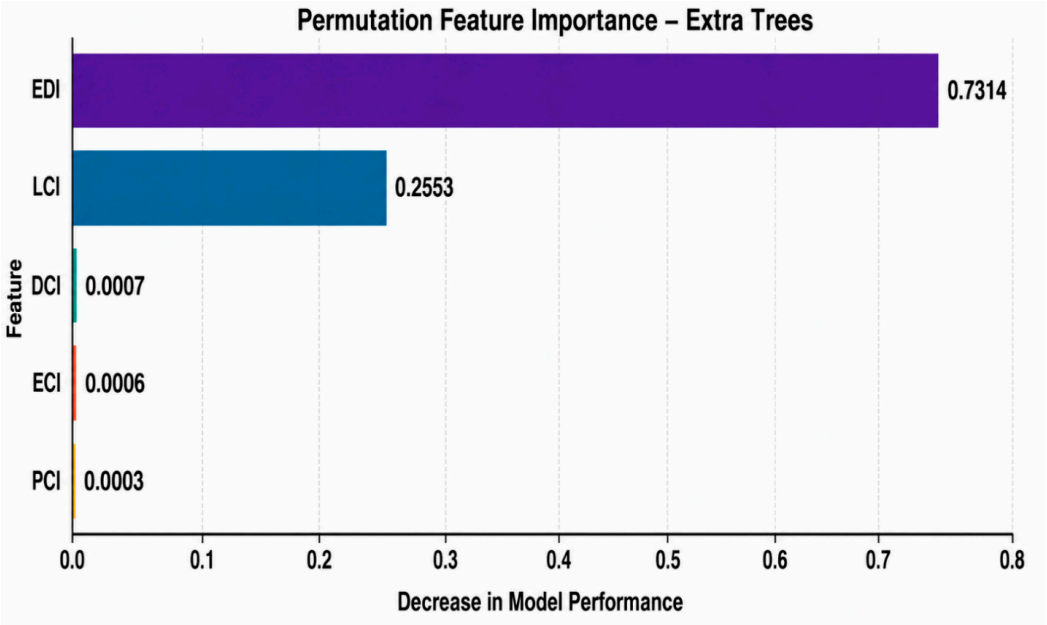

Figure 19. Permutation Feature Importance Analysis.

The permutation results presented in Figure 19 further confirms these results. The disturbance of EDI contributed the most to the loss in predictive performance (0.7314), followed by LCI (0.2553). On the other hand, DCI (0.0007), ECI (0.0006), and PCI (0.0003) barely affected the predictive power of the model.

In sum, the analysis of explainable machine learning confirms and complements the cluster, robustness, external validity, and entropy-based complexity analyses in terms of the relative informational significance of democratic dimensions in the context of the V-Dem measurement scheme. It is evident that the consistency of SHAP, LIME, and permutation importance implies that the electoral and liberal institutional dimensions have the highest predictive significance for LDI variation. Nevertheless, these results have to be understood as predictive associations, not causation, considering the conceptual connection between LDI and its components.

5. Discussion

The analysis demonstrates that democracy can better be understood as a multidimensional institution rather than a single dimensional continuum. Despite the use of a democracy index as a measure on a single continuum, the main K-means categorization demonstrates several unique combinations of electoral, liberal, participatory, deliberative, and egalitarian institutions in different countries. Therefore, countries that share common levels of democratic development may significantly differ in institutional structures.

The robustness analysis additionally reveals more information about the stability of the structure that has been detected. At the same time, the GMM estimates prove the existence of gray zones between regimes. It means that the democratic development process is not a process of overcoming one institutional structure and moving to the next one but rather a process of developing new institutional structures simultaneously with other ones.

Thus, the choice of seven clusters should not be viewed as a proof of seven being the only optimal number of democracy regimes. Various criteria for clustering yielded different solutions due to the complex multidimensional nature of the data. A seven-cluster K-means solution was adopted as the preferred classification due to the substantive interpretability of the resulting partitions as well as preservation of meaningful differentiation between the

institutional arrangements. The GMM model was applied as a robustness check and not as an alternative to the adopted classification.

Robustness evidence must be considered in the context of the elements analyzed using each of the three tests. The bootstrap and consensus analyses tested for the replicability of cluster assignments, while the GMM test looked at the agreement between different clustering methods. These tests do not prove that the same substantive conclusions will follow from a change in the number of clusters used in the analysis. Likewise, the Complexity Index based on the measure of entropy relies on cutoff points used to transform democracy indicators into low, medium, and high levels. While the threshold points at 1/3 and 2/3 seem to be obvious, the results currently obtained do not show that entropy levels within regimes or their rankings will not change using alternative cutoff points. Hence, the seven-cluster classification and measures of complexity based on entropy are viewed conditionally rather than uniquely.

The entropy method also provides another perspective on the problem in question, involving a quantification of the heterogeneity of regimes in terms of democracy variables. More specifically, the Complexity Index represents the average entropy of the distributions of countries according to predefined institutional states for every democracy variable in each regime. High entropy implies a high heterogeneity of the distribution of countries across institutional states while low entropy denotes a smaller number of states where these countries concentrate. The research has demonstrated that the regimes are characterized not only by differences in their institutional profiles but also in the heterogeneity of cross-country distribution of institutional states as well. Indeed, a high entropy of hybrid and intermediate regimes means a high heterogeneity of countries within them, measured in terms of each democracy variable, while the low entropy of more consolidated regimes implies a high concentration of countries in a small number of institutional states. These entropies cannot be treated as a measure of institutional balance in each individual country or as a measure of dependence or correlation between democratic dimensions.

Additionally, the external validation through the World Justice Project yields independent confirmation that the primary K-means regimes exhibit systematic differences in terms of the rule of law outcomes. Since the WJP indicators were not part of the construction of the regimes, the differences noted here represent external validation of the regimes and not the clustering exercise itself. However, the findings should still be seen as evidence of associations and not as evidence of causation.

A crucial caveat pertains to the additional predictive modeling of the LDI. Given that the LDI index consists of the two other indexes, EDI and LCI, according to the predetermined aggregation method, the extremely high predictability and high feature importance of such models are in some ways to be expected. Hence, these results cannot be viewed as independent evidence of newly established institutional relationships. Rather, the comparison of these models with the initial V-Dem aggregation formula serves as a baseline for evaluating the degree of reproduction of the predefined index. This way, the substantial meaning of the predictive results can be limited, whereas their descriptive value will remain unchanged. Consequently, the main value of the present work does not consist of predicting the index based on its components, but rather in the multidimensional analysis of institutions, their robustness, and the heterogeneity inside regimes.

In sum, the findings support a multidimensional understanding of democracy where states have different combinations and levels of institutional development. Clustering, entropy, robustness, and external validation provide a methodology through which we can explore differences between and within regimes.

6. Policy Implications

These findings have some important implications for the study of comparative democracy and institutional analysis. First, the existence of different institutional arrangements implies that even countries with very similar democratic scores can have different mixes of strong and weak institutions. For comparative analyses, it is useful to consider both democratic components and overall democracy scores.

Second, the entropy results show that institutional diversity differs by regime type. It means that the analysis of democratic development should include the consideration not only of average institutional performance but also of institutional coherence.

The third point is that the suggested typology might be used as an extra analytical tool for comparisons of democratic development and institutional evolution. Researchers and international agencies will be able to determine the similarities between countries in terms of their multidimensional institutional structure and to study how they change over time. The use of the typology, nonetheless, should be limited to its analytical function only.

7. Conclusions

This study employs a multidimensional approach to classifying global democracy regimes based on the use of six democracy dimensions according to V-Dem. The main K-means analysis results in seven distinct institutional regimes. From the findings, it is evident that there are variations between countries in terms of levels of democratization as well as in terms of institutional setups.

From the entropy analysis, we learn of variations in institutional heterogeneities among the different regimes. Additionally, GMM analysis was used to validate the findings. On the other hand, the rule-of-law analysis conducted independently by the WJP has revealed systematic variations among the regimes.

The benefit of this framework is that it brings together unsupervised machine learning and information-theory analysis with independent validation. Instead of looking at democracy as a one-dimensional concept, the framework allows the analysis of multidimensional institutional configurations and their heterogeneity. Further longitudinal analysis using this framework might involve analyzing how countries move from one institutional configuration to another over time.

8. Limitations and Future Research

However, there are several limitations that should be pointed out. First, the basic analysis uses cross-sectional V-Dem data for 2024 and thus does not consider any temporal dynamics or regime transformations.

Second, the classification is dependent upon the choice of V-Dem dimensions as well as on the procedure of clustering. Even though a series of diagnostics and robustness checks have been undertaken, the findings do not prove that the seven-cluster solution is the only optimal one or that the substantive meanings will be invariant with respect to other choices of the number of clusters. Moreover, the Complexity Index based on the entropy measure is sensitive to the cutoff points used to define low, medium, and high levels of the democracy variables. Inasmuch as this analysis relies on the cutoff points of 1/3 and 2/3, the sensitivity of the entropy values and regime classifications to the other choices of cutoff points remains an empirical issue.

Third, the validation of results is performed using the subsample of those countries where the V-Dem and WJP data can be matched. However, although the subsample provides an independent source of validation, it is not representative of all V-Dem countries. Finally, the current study reveals only empirical links, not causes.

Further research might consider the use of this methodology in applying V-Dem data longitudinally to study regime change and institutional stability. Further validation of the regimes identified might be done by conducting research based on alternative democracy, governance, and institutional datasets. Time-sensitive analysis as well as dynamic clustering/classification might allow for the study of the dynamics of institutional configurations in time and space.

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