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Integral Formulas for Hermite-Based Peters-Type Simsek Polynomials and Their Applications

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Abstract

Integrals and derivatives serve as fundamental instruments across virtually all scientific disciplines. Due to their extensive application, researchers have consistently sought to develop novel identities and formulas for both operations. This study aims to introduce several new integral formulas involving Hermite-based Peters-type Simsek polynomials, alongside their associated functions. These results establish connections with logarithmic functions and various special sequences, including bivariate Hermite polynomials and Peters-type Simsek numbers. Additionally, the paper provides insightful remarks and observations regarding the finding.

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1. INTRODUCTION

The generating functions play a fundamental role in the study of special numbers and polynomials, with applications in fields such as mathematical physics, engineering, and applied mathematics. These functions are a useful tool for representing sequences. Many special numbers can be defined using this method. Furthermore, this approach makes calculations easier. Generating functions help to examine mathematical structures more clearly. Therefore, studying these types of functions is important. (Banderier et al., 2002; Yuluklu, 2018a; Yuluklu, 2018b; Yuluklu, 2023; Yuluklu, 2024, Simsek, 2019a; Simsek, 2019b; Simsek, 2020;Simsek, 2021). In this study, Hermite-based Peters-type Simsek polynomials are investigated. Many new integral formulas are given for these polynomials.

Before presenting these results, let's provide some definitions:

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In 2018, Simsek (2018a,2018b) utilized p-adic integrals over the set of p-adic integers to construct novel generating functions for the numbers $Y_{n,u}(\varepsilon)$:

$$\frac{2\sum_{j=0}^{f-1}(-1)^j\chi(j)\varepsilon^j(1+\varepsilon t)^j}{\varepsilon^f(1+\varepsilon t)^f-1}=\sum_{n=0}^{\infty}\frac{Y_{n,u}(\varepsilon)}{n!}t^n, \tag{1}$$

where x represents the Dirichlet character (Simsek, 2018a) and (Simsek,2018b) (page 568)).

Using the motivation for the generating function presented in (1), Simsek defined the following numbers:

$$f_Y(t,\varepsilon)=\frac{2}{\varepsilon(1+\varepsilon t)-1}=\sum_{n=0}^{\infty}\frac{Y_n(\varepsilon)}{n!}t^n, \tag{2}$$

where $\varepsilon(1+\varepsilon t)\neq 1$ and $\varepsilon\epsilon\mathbb{R}$ (or $\mathbb{C}$) (Simsek, 2018a) and (Simsek, 2018b) (Equation (2.13))). The numbers $Y_n(\varepsilon)$ are known as the first-kind Peters-type Simsek numbers.

The first-kind Peters-type Simsek polynomials, denoted $Y_n(\omega;\varepsilon)$, are given by

$$f_Y(t,\omega;\ \varepsilon)=\frac{2(1+\varepsilon t)^{\omega}}{\varepsilon(1+\varepsilon t)-1}=\sum_{n=0}^{\infty}\frac{Y_n(\omega;\varepsilon)}{n!}t^n, \tag{3}$$

where $\varepsilon(1+\varepsilon t)\neq 1$ and $\lambda\epsilon\mathbb{R}$ (or $\mathbb{C}$) (Simsek, 2018a) and (Simsek,2018b) (Equation (2.19))).

When $\omega=0$ in (3), we have

$$Y_n(0;\varepsilon)=Y_n(\varepsilon).$$

The numbers $Y_n(\varepsilon)$ are computed by

$$Y_n(\varepsilon)=2(-1)^n\frac{n!}{\varepsilon-1}\left(\frac{\varepsilon^2}{\varepsilon-1}\right)^n$$

(Simsek, 2018a) and (Simsek, 2018b) (Equation (2.18))).

The numbers $Y_n(\omega;\varepsilon)$ are computed by

$$Y_n(\omega;\varepsilon)=\sum_{k}^{n}\binom{n}{k}\varepsilon^{n-k}\binom{\omega}{n-k}(n-k)!\,Y_k(\varepsilon)$$

(Simsek, 2018a) and (Simsek,2018b), (Equation (2.20))).

The Peters polynomials $S_n(\omega;\varepsilon,\mu)$ are given by

$$\frac{(1+t)^{\omega}}{(1+(1+t)^{\lambda})^{\mu}}=\sum_{n=0}^{\infty}\frac{S_n(\omega;\varepsilon,\mu)}{n!}t^n \tag{4}$$

(Roman, 2005).

Drawing on the structural framework established by Kucukoglu et al. (2019) and Kucukoglu (2024), the higher-order generalizations for the first-kind Peters-type Simsek numbers and polynomials, $Y_n^{(k)}(\varepsilon)$ and $Y_n^{(k)}(\omega;\varepsilon)$ are defined through the following generating functions, respectively:

$$\left(\frac{2}{\varepsilon^2 t+\varepsilon-1}\right)^{k}=\sum_{n=0}^{\infty}\frac{Y_n^{(k)}(\varepsilon)}{n!}t^n \tag{5}$$

and

$$\left(\frac{2}{\varepsilon^2 t+\varepsilon-1}\right)^{k}(1+\varepsilon t)^{\omega}=\sum_{n=0}^{\infty}\frac{Y_n^{(k)}(\omega;\varepsilon)}{n!}t^n \tag{6}$$

in which the condition $\varepsilon \in \mathbb{R}$ (or $\mathbb{C}$) with maintained for a positive integer *k* (Kucukoglu et al., 2019) Equations (2.1) and (2.2); see also Kucukoglu (2024).

Setting $k=1$ in (5) and (6), we easily reduce them to (2) and (3). That is,

$$Y_n^{(1)}(\varepsilon)=Y_n(\varepsilon)$$

and

$$Y_n^{(1)}(\omega;\varepsilon)=Y_n(\omega;\varepsilon).$$

Utilizing (1), for $f \equiv 1$, this equation reduced to the equation (1) (Simsek, 2018a; Simsek, 2018b). Recently, equations (2) and (3) have found numerous applications. For instance, Khan et al. (2020; 2022) identified these as Simsek polynomials and introduced their two-variable counterparts. Building on these developments, Kucukoglu (Kucukoglu and Simsek, 2021; Kucukoglu,2024) generalized these concepts, naming them Peters-type Simsek numbers and polynomials. Subsequent to equations (5) and (6), Kucukoglu further extended the Hermite-based Simsek sequences (Kucukoglu, 2023).

The two-variable Hermite polynomials are defined by

$$f_H(t,u,v,j) = e^{ut+vt^j} = \sum_{n=0}^{\infty} \frac{H_n{}^{(j)}(u,v)}{n!} t^n, \tag{7}$$

where $j \geq 2$ (Banderier et al., 2002; Yuluklu, 2018a; Yuluklu, 2018b; Yuluklu, 2023; Yuluklu, 2024, Simsek, 2019a; Simsek, 2019b; Simsek, 2020;Simsek, 2021).

The history of Hermite polynomials can be traced to the work of Pierre-Simon Marquis de Laplace in 1811 (Laplace, 1811). Since then, these polynomials have been characterized in various forms. A standard definition for probabilistic Hermite polynomials is given as follows:

A standard definition for probabilistic Hermite polynomials is given as follows:

$$H(\omega,k) = (-1)^k exp\left(\frac{\omega^2}{2}\right) D^k\left(e^{-\frac{\omega^2}{2}}\right), \tag{8}$$

where $exp(\omega) = e^{\omega}$ and $D^k = \frac{d^k}{d\omega^k}$ (Banderier et al., 2002;Yuluklu. 2018a; Yuluklu, 2018b; Yuluklu, 2023; Yuluklu, 2024, Simsek, 2019a; Simsek, 2019b; Simsek, 2020;Simsek, 2021).

The polynomials $H(\omega,k)$ satisfy the Strum-Liouville differential equations below:

$$D\left(e^{-\frac{1}{2}\omega^2} y'\right) + \alpha e^{-\frac{1}{2}\omega^2} y = 0, \tag{9}$$

where α is a constant and $y' = \frac{dy}{d\omega}$ (Hermite, 1864).

Based on equations (8) and (9), it is widely recognized that Hermite polynomials are indispensable in physics, where their governing differential equations are typically expressed as:

$$y'' - 2xy' + 2\alpha y = 0 \tag{10}$$

(Hermite, 1864). The theory of differential equations offers numerous applications for equation (1). Here we note that, in the present study, we do not focus on the solutions to these equations, as they have been extensively documented in existing literature (Banderier et al., 2002; Yuluklu,2018a; Yuluklu, 2018b; Yuluklu,2023; Yuluklu, 2024, Simsek, 2019a; Simsek, 2019b; Simsek, 2020;Simsek, 2021).

In 2020, Yuluklu introduced the following generating functions:

$$G_1(t;u,v,j) = f_Y(t,\varepsilon) f_H(t,u,y,j)$$

and

$$G_2(t;u,v,\omega;\varepsilon,j) = f_Y(t,\omega;\varepsilon) f_H(t,u,v,j)$$

(Yuluklu, 2020 (Equations (4) and (5)). That is, the Hermite-type combinatorial Simsek polynomials are expressed as:

$$G_1(t;u,v,j) = \frac{2e^{ut+vt^j}}{\varepsilon(1+\varepsilon t)-1} = \sum_{n=0}^{\infty} \frac{{}_Y H_n{}^{(j)}(u,v;\varepsilon)}{n!} t^n \tag{11}$$

and

$$G_2(t;u,v,\omega;\varepsilon,j) = \frac{2(1+\varepsilon t)^{\omega} e^{ut+vt^j}}{\varepsilon(1+\varepsilon t)-1} = \sum_{n=0}^{\infty} \frac{{}_Y H_n(u,v,\omega;\varepsilon,j)}{n!} t^n, \tag{12}$$

where $\varepsilon(1+\varepsilon t) \neq 1$ (Yuluklu, 2020 (Equations (4) and (5)).

From (11) and (12), one can easily see

$${}_Y H_n{}^{(j)}(0,0;\varepsilon) = Y_n(\varepsilon)$$

and

$${}_Y H_n(0,0,\omega;\varepsilon,j) = Y_n(\omega;\varepsilon).$$

2. MAIN RESULTS

Here, using generating functions of the polynomials ${}_{Y}H_{n}^{(j)}(u,v)$ and the polynomials ${}_{Y}H_{n}(u,v,\omega;\varepsilon,j)$, some integral formulas are given related to these polynomials.

Upon performing an integration of (11) with respect to y from 0 to a and executing the required algebraic steps, we find:

$$\int_{0}^{a} G_{2}(t;u,v,\omega;\varepsilon,j)\,dv = \frac{2(1+\varepsilon t)^{\omega}e^{ut}\left(e^{at^{j}}-1\right)}{t^{j}(\varepsilon(1+\varepsilon t)-1)}, \tag{13}$$

where $t \neq 0$, $\varepsilon(1+\varepsilon t) \neq 1$. It is important to observe that (13) provides a distinct generating function, which leads us to:

$$\int_{0}^{a} G_{2}(t;u,v,\omega;\varepsilon,j)\,dv = f_{y}(t,\omega;\varepsilon)\,e^{ut}\frac{e^{at^{j}}-1}{t^{j}}. \tag{14}$$

From the relationship established in (14), we state the following theorem:

Theorem 1.

$$\textstyle\int_{0}^{a} G_{2}(t;u,v,\omega;\varepsilon,j)\,dv = \frac{1}{t^{j}}\left(G_{2}(t;u,a,\omega;\varepsilon,j) - G_{2}(t;u,0,\omega;\varepsilon,j)\right). \tag{15}$$

From (13), one has

$$\int_{0}^{a} G_{2}(t;u,v,\omega;\varepsilon,j)\,dv = \frac{1}{t^{j}}\sum_{n=0}^{\infty}\frac{t^{n}}{n!}\left({}_{Y}H_{n}(u,a,\omega,\varepsilon,j)\right) - {}_{Y}H_{n}(u,0,\omega,\varepsilon,j)$$

$$= \sum_{n=0}^{\infty}\left(\left({}_{Y}H_{n}(u,a,\omega,\varepsilon,j)\right) - {}_{Y}H_{n}(u,0,\omega,\varepsilon,j)\right)\frac{t^{n-j}}{n!}.$$

Thus, we get

$$\int_{0}^{a}\sum_{n=0}^{\infty}{}_{Y}H_{n}\,(u,v,\omega;\varepsilon,j)\,\frac{t^{n}}{n!}\,dv = \sum_{n=0}^{\infty}\frac{{}_{Y}H_{n+j}\,(u,a,\omega,\varepsilon,j\,)}{\binom{n+j}{j}\,j!} - \frac{{}_{Y}H_{n+j}(u,0,\omega,\varepsilon,j\,)}{\binom{n+j}{j}\,j!}\frac{t^{n}}{n!},$$

where interchange is valid within radius of convergence. By equating the coefficients associated with $\frac{t^n}{n!}$ in the preceding formula, we establish the following theorem:

Theorem 2.

$$\int_0^a {}_Y H_n(u,v,\omega;\varepsilon,j)\,dv = \frac{{}_Y H_{n+j}(u,a,\omega,\varepsilon,j) - {}_Y H_{n+j}(u,0,\omega,\varepsilon,j)}{\binom{n+j}{j} j!}.$$

When $a = 1$ in the Theorem 2, we obtain the following result:

Corollary 3.

$$\int_0^1 {}_Y H_n(u,v,\omega;\varepsilon,j)\,dv = \frac{{}_Y H_{n+j}(u,1,\omega,\varepsilon,j) - {}_Y H_{n+j}(u,0,\omega,\varepsilon,j)}{\binom{n+j}{j} j!}. \tag{16}$$

Evaluating the integral of (12) with respect to *u*, one has

$$\int_0^a G_2(t;u,v,\omega;\varepsilon,j)\,du = \frac{2(1+\varepsilon t)^{\omega} e^{vt^j}(e^{at}-1)}{t(\varepsilon(1+\varepsilon t)-1)}.$$

Thus

$$\int_0^a G_2(t;u,v,\omega;\varepsilon,j)\,du = f_y(t,\omega;\varepsilon)\, e^{vt^j}(e^{at}-1).$$

From the relationship established in the above equation, we state the Theorem 4:

Theorem 4.

$$\int_0^a G_2(t;u,v,\omega;\varepsilon,j)\,du = \frac{1}{t}\big(G_2(t;a,v,\omega;\varepsilon,j) - G_2(t;0,v,\omega;\varepsilon,j)\big). \tag{17}$$

By applying (17) to (12), we get

$$\sum_{n=0}^{\infty}\int_{0}^{a} {}_{Y}H_{n}(u,v,\omega;\varepsilon,j)du\ \frac{t^{n}}{n!}$$
$$=\sum_{n=0}^{\infty} {}_{Y}H_{n}(a,v,\omega;\varepsilon,j)\frac{t^{n-1}}{n!}-\sum_{n=0}^{\infty} {}_{Y}H_{n}(0,v,\omega;\varepsilon,j)\frac{t^{n-1}}{n!}$$

Thus, we have

$$\sum_{n=0}^{\infty}\int_{0}^{a}({}_{Y}H_{n}(u,v,\omega;\varepsilon,j)du)\ \frac{t^{n}}{n!}$$
$$=\sum_{n=0}^{\infty}\frac{{}_{Y}H_{n+1}(a,v,\omega,\varepsilon,j)-{}_{Y}H_{n+1}(0,v,\omega,\varepsilon,j)}{n+1}\frac{t^{n}}{n!}.$$

A comparison of the components in the above expression results in the theorem provided below:

Theorem 5.

$$\int_{0}^{a} {}_{Y}H_{n}(u,v,\omega;\varepsilon,j)du=\frac{{}_{Y}H_{n+1}(a,v,\omega,\varepsilon,j)-{}_{Y}H_{n+1}(0,v,\omega,\varepsilon,j)}{n+1}. \tag{18}$$

When $a=1$ in the Theorem 5, we obtain the following result:

Corollary 6.

$$\int_{0}^{1} {}_{Y}H_{n}(u,v,\omega;\varepsilon,j)du=\frac{{}_{Y}H_{n+1}(1,y,\omega,\varepsilon,j)-{}_{Y}H_{n+1}(0,y,\omega,\varepsilon,j)}{n+1}.$$

Joining (12) with (7), we have

$${}_{Y}H_{n}(u,v,\omega;\varepsilon,j)=\sum_{m=0}^{n}\binom{n}{m}Y_{n-m}(\omega;\varepsilon)H_{m}^{(j)}(u,v) \tag{19}$$

(Yuluklu, 2020 (Theorem 1)). Further since

$$H_m^{(j)}(u,v) = \sum_{s=0}^{\left[\left[\frac{m}{j}\right]\right]} \frac{m!\, u^{m-js} v^s}{(m-js)!\, s!},$$

where $\left[\left[\frac{m}{j}\right]\right]$ is the largest integer $\leq \frac{m}{j}$, we obtain

$${}_Y H_n(u,v,\omega;\varepsilon,j) = \sum_{m=0}^{n} \binom{n}{m} Y_{n-m}(\omega;\varepsilon) \sum_{s=0}^{\left[\left[\frac{m}{j}\right]\right]} \frac{u^{n-js} v^s}{(n-js)!\, s!}. \tag{20}$$

If we integrate equation (20) relative to x from 0 to a, we reach the Theorem 7:

Theorem 7.

$$\int_0^a {}_Y H_n(u,v,\omega;\varepsilon,j)du = \sum_{m=0}^{n} \binom{n}{m} Y_{n-m}(\omega;\varepsilon) \sum_{s=0}^{\left[\left[\frac{m}{j}\right]\right]} \frac{v^s a^{n-js+1}}{s!\,(n-js)!\,(n-js+1)}. \tag{21}$$

By merging the identities in (18) and (21), a novel formula is obtained:

Corollary 8.

$$\sum_{m=0}^{n} \binom{n}{m} Y_{n-m}(\omega;\varepsilon) \sum_{s=0}^{\left[\left[\frac{m}{j}\right]\right]} \frac{v^s a^{n-js+1}}{s!\,(n-js+1)!} = \frac{{}_Y H_{n+1}(a,v,\omega;\varepsilon,j) - {}_Y H_{n+1}(0,v,\omega;\varepsilon,j)}{n+1}.$$

Now, we give some applications of the above results:

Firstly we give some numerical values of the polynomials ${}_Y H_n(u,v,\omega;\varepsilon,j)$.

By using (19), for $j = 1$ and $n = 0,1,2$, the values of the polynomials ${}_Y H_n(u,v,\omega;\varepsilon,1)$ are given:

$${}_Y H_0(u,v,\omega;\varepsilon,1) = \frac{2}{\varepsilon - 1},$$

$${}_Y H_1(u,v,\omega;\varepsilon,1) = \frac{2(u+v)}{\varepsilon-1} - \frac{2\varepsilon^2}{(\varepsilon-1)^2} + \frac{2\varepsilon\omega}{\varepsilon-1},$$

$${}_Y H_2(u,v,\omega;\varepsilon,1) = \frac{2(u^2+2uv+v^2)}{\varepsilon-1} + \frac{4\varepsilon^4}{(\varepsilon-1)^3} - \frac{4\varepsilon\omega}{(\varepsilon-1)^2}$$

$$+\frac{2\varepsilon^2(\omega-1)\omega}{\varepsilon-1}+2(u+v)\left(-\frac{2\varepsilon^2}{(\varepsilon-1)^2}+\frac{2\varepsilon\omega}{\varepsilon-1}\right).$$

By using (19), for $j=2$ and $n=0,1,2$, the values of the polynomials ${}_YH_n(u,v,\omega;\varepsilon,2)$ are given:

$${}_YH_0(u,v,\omega;\varepsilon,2)=\frac{2}{\varepsilon-1},$$

$${}_YH_1(u,v,\omega;\varepsilon,2)=\frac{2u}{\varepsilon-1}-\frac{2\varepsilon^2}{(\varepsilon-1)^2}+\frac{2\varepsilon\omega}{\varepsilon-1},$$

$${}_YH_2(u,v,\omega;\varepsilon,2)=\frac{2(u^2+2v)}{\varepsilon-1}+\frac{4\varepsilon^4}{(\varepsilon-1)^3}-\frac{4\varepsilon^3\omega}{(\varepsilon-1)^2}$$
$$+\frac{2\varepsilon^2(\omega-1)\omega}{\varepsilon-1}+2u\left(-\frac{2\varepsilon^2}{(\varepsilon-1)^2}+\frac{2\varepsilon\omega}{\varepsilon-1}\right).$$

By using (19), for $j=3$ and $n=0,1,2$, the values of the polynomials ${}_YH_n(u,v,\omega;\varepsilon,3)$ are given:

$${}_YH_0(u,v,\omega;\varepsilon,3)=\frac{2}{\varepsilon-1},$$

$${}_YH_1(u,v,\omega;\varepsilon,3)=\frac{2u}{\varepsilon-1}-\frac{2\varepsilon^2}{(\varepsilon-1)^2}+\frac{2\varepsilon\omega}{\varepsilon-1},$$

$${}_YH_2(u,v,\omega;\varepsilon,3)=\frac{2u^2}{\varepsilon-1}+\frac{4\varepsilon^4}{(\varepsilon-1)^3}-\frac{4\varepsilon^3\omega}{(\varepsilon-1)^2}$$
$$+\frac{2\varepsilon^2(\omega-1)\omega}{\varepsilon-1}+2u\left(-\frac{2\varepsilon^2}{(\varepsilon-1)^2}+\frac{2\varepsilon\omega}{\varepsilon-1}\right).$$

Secondly we give numerical applications the Corollary 3:

When $n=j=1$ in (16), we obtain

$$\int_0^1 {}_YH_1(u,v,\omega;\varepsilon,1)\,dv=\frac{{}_YH_2(u,1,\omega,\varepsilon,1)-{}_YH_2(u,0,\omega,\varepsilon,1)}{2}.$$

From the above calculations, we get

$$\int_0^1 {}_Y H_1(u,v,\omega;\varepsilon,1)\,dv = \frac{-1+2u(\varepsilon-1)+\varepsilon+2\varepsilon^2(\omega-1)-2\varepsilon\omega}{(\varepsilon-1)^2}.$$

3. RESULTS AND DISCUSSION

The formulas presented in this study involve finite sums originated from generating functions and integral representations of Hermite-based Peters-type, bivariate Hermite, and Peters-type Simsek polynomials. These mathematical frameworks are expected to be utilized in them theoretical and applied sciences.

This research focuses on the derivation of novel identities, including integral formulas and finite sums, by utilizing the aforementioned generating functions. The scope of the present results extends to two-variable Hermite polynomials, Peters-type Simsek polynomials, and Hermite-based Peters-type structures. The significance of Hermite polynomials is well-documented in fields such as quantum mechanics particularly in the context of probability theory, statistical physics, quantum mechanics, and differential equations, due to their powerful generating functions, integral representations, and orthogonality properties. While two-variable Hermite polynomials, as a multivariate generalization of this classical structure, have yielded important results in partial differential equations, heat conduction models, and combinatorial expansions. There are many studies in the literature on this type of numbers and polynomials. Authors have investigated the properties of these types of polynomials and have provided results for them, see (Khan et al., 2020; Yuluklu, 2018a; Yuluklu, 2018b; Yuluklu, 2023; Yuluklu, 2024, Simsek, 2019a; Simsek, 2019b; Simsek, 2020;Simsek, 2021). This study also utilizes the generating function approach to present finite sum identities, integral formulas, and closed-form evaluations of special values.

4. CONCLUSION

This paper has introduced integral formulas and finite sums specifically for Hermite-based Peters-type Simsek polynomials. Our results demonstrate clear linkages with bivariate Hermite polynomials and other Peters-type sequences. Our future research aims to explore the partial and ordinary differential equations (PDEs and ODEs) associated with these polynomials through their generating functions.

In essence, the comprehensive analysis of Peters-type Simsek and Hermite polynomials presented in this work offers new perspectives on their functional behaviors. By establishing these derivative and integral relations, the paper enriches the existing literature and holds the potential to impact diverse scientific fields, ranging from statistical probability to complex differential modeling.

AUTHOR CONTRIBUTIONS

AI Exclusion: AI tools are strictly prohibited from being listed as authors.

Legal Consent: All human authors confirm they have reviewed and legally accepted the final version under the **CC BY-SA 4.0** framework.

CONFLICT OF INTEREST

The author declares no conflicts of interest.

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